THE EVOLUTION OF INCOME AND FERTILITY DISTRIBUTIONS OVER THE COURSE OF ECONOMIC DEVELOPMENT: A HUMAN CAPITAL PERSPECTIVE*

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ABSTRACT

We model the evolution of income and fertility distributions over the course of economic development using an endogenous-growth framework where human capital is the engine of both income growth and income distribution. In our OLG setting, heterogeneous families determine fertility and human capital formation in children, and generations are linked through intra-family and inter-family interactions. We conduct simulations and regression analyses to test propositions concerning the behavior of inequalities in fertility and schooling attainments, as well as of three income inequality measures – family-income inequality, income-group inequality, and the Gini coefficient – over three phases of economic development. In this context, we also reexamine the "Kuznets hypothesis" concerning the relation between income inequality and income growth.

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I. INTRODUCTION

Using historical data from a number of developing and developed countries, Kuznets (1955, 1963) argued that income inequality first rises and then falls during a transitional development period. The literature following Kuznets developed in two main directions. The first tested his hypothesis against empirical data from many other countries.[1] The second dealt with the development–inequality nexus theoretically as a causal relation going from either growth to inequality or vice versa.[2] Both sets of studies have offered conflicting conclusions about the competing theoretical and empirical hypotheses.

One issue that received less attention in this literature is the comparative income inequality levels in the two phases of economic development that frame the transition phase, which we model in this paper as "stagnant equilibrium" and "growth equilibrium" steady states. Other observations also received little attention so far. For example, our data in section V reveal a positive correlation between inequality in income and in schooling attainments. Furthermore, historical data indicate that fertility differences across income groups tend to be attenuating in the pre-takeoff phase as well as in advanced phases of development, while expanding over the transitional phase (see Coale and Treadway, 1986).

We attempt to bring together this broader evidence by developing a deterministic, endogenous-growth model with heterogeneous families, in which human capital is the key asset determining growth and the distribution of income, and families determine fertility and educational investments. Our model offers a dynamic extension of Becker's (1967) deterministic model of income distribution, as well as a generalization of more recent work by Ehrlich and Lui (1991), Tamura (1991), Zhong (1998), and Ehrlich and Yuen (2000). We show that the behavior of income inequality over the transitional development phase can vary across different countries, depending on the factors triggering the economic takeoff and the income-inequality measure used. Moreover, we offer new insights about the dynamic evolution of observed inequalities across families not just in income, but also in fertility and human capital investments over the course of development, including the pre- and post-transition phases.

Our model suggests that the relationship between income growth and income inequality is **associative**, not causal. Three main forces influence this association: a. interactions between overlapping generations within families, which enable continuous human capital formation in successive generations; b. heterogeneities in endowments and investment efficiencies across families, which affect the degree of inequality across families; and c. social interactions, which help attaining stable dynamic equilibria in the distributions of both income and fertility.[3]

Formally, we set up an OLG model of endogenous growth with finitely-lived individuals. Parents optimize on investments in the quantity and quality of children (in an extended version, savings can be introduced as well). Heterogeneities across families and social interactions enable us to derive equilibrium paths of income, schooling, and fertility distributions over three phases: a stagnant steady state, a perpetual-growth steady state, and a transition phase linking the two.

By this approach we are also able to provide new insights about the "Kuznets hypothesis". The observed association between the level and inequality of income is influenced partly by the factors determining the comparative inequality levels at the growth vs. stagnant steady state. The association is in a state of flux during the transitional phase. Specifically, we show that over this phase the income inequality path can assume a U shape, an inverted-U shape, or combinations of the two, with the inequality level ultimately rising, falling or staying the same, depending on the way heterogeneity sources are correlated across families, how different takeoff-triggers affect different family groups, and the inequality measure used. We derive three such measures as endogenous variables: family-income inequality, income-group inequality, and the Gini coefficient. A major inference of our model is that regardless of the shape of the income inequality path over the transitional phase, the inequality paths of fertility and human capital investments are expected to exhibit an inverted-U shape with both tails converging on equality.

Section II introduces the model and its equilibrium solutions. In section III we derive equilibrium regimes and comparative dynamic implications, and in section IV we present simulated dynamic paths of our inequality measures. Section V then presents new evidence on fertility, schooling, and income distributions based on international panel data from 1950-98.

II. THE MODEL AND EQUILIBRIUM SOLUTIONS

A. The Economic Environment

To derive income and fertility inequality paths over the entire development process, we extend the deterministic representative-family, OLG model of endogenous growth in Ehrlich and Lui [EL] (1991) to a heterogeneous-family case that recognizes inter-family interactions as well.

The Economy: The economy is comprised of a fixed distribution of heterogeneous family groups of varying earning capacities, indexed in a decreasing order by i (i = 1, 2, 3…I). We implicitly rely on positive assortative mating within groups by their distinct sources of heterogeneity to maintain this fixed distribution over time, because if inter-group mating is allowed, and children inherit the average characteristics of their parents, human capital attainments would eventually converge in all families.[4] Each agent in this economy lives through three periods: childhood (t-1), adulthood (t), and old age (t+1). All family-based decisions are made by young parents.

Like Becker (1967), we focus on three objective sources of inherited heterogeneity: a. differences in ability (A^i); b. differences in income-yielding "endowments" ($\overline{H}^i$), stemming from inherited social status, political power, or other personal assets; c. differences in education-financing costs (θ^i). We generally do not allow for differences in preferences or external production technologies, since these need not be related to objective personal differences.

Goods Production and Income: The economy is competitive and human capital is the sole asset. A young parent in family-group i possesses a production capacity ($\overline{H}^i+H^i_t$), composed of a fixed inherited income-producing endowment ($\overline{H}^i$), measured in units of human capital, and an acquired human capital stock, (H^i_t), attained through parental inputs. Labor supply by each young parent is fixed in any period. We also assume for convenience that all consumer goods, including educational services, can be purchased. Under a linear and strongly additive production technology for all goods, aggregate income equals aggregate labor employment, or families' actual production capacities in each period (see equation 1), Y = L, and the competitive firms' zero-profit condition, $\pi=Y-\varpi L=0$, yields a time-invariant real rental rate per unit of labor, $\varpi=1$, which also guarantees full employment. We initially abstract from any savings, so earnings are identical to income. In **Appendix A**, we allow for savings and show that our inferences concerning earnings inequality extend to income inequality as well.

Human-capital production: The human-capital formation rule is given by:

(1) $H^i_{t+1} = A^i h^i_t(\overline{H}^i+H^i_t)^{1-\gamma}[(\overline{H}^1+H^1_t)(N^1_t/N^i_t)]^\gamma \equiv A^i h^i_t(\overline{H}^i+H^i_t)(S^i_t)^\gamma$,

where h^i_t is the share of production capacity a young parent from family-group i (i = 1, 2,…I) invests in educating each child, $\overline{H}^i$ and H^i_t are the parent's respective endowed and attained human capital stocks, and N^1_t/N^i_t is the ratio of the population shares of parents in family groups 1 and i. S^i_t denotes an inter-family, "social interaction" factor. Note that equation (1) becomes $A^1h^1_t(\overline{H}^1+H^1_t)$ if i = 1.

Equation (1) aims to capture two types of interactions within and across families: a. Persistent human capital formation can be sustained over time only if the older generation of parents invests in the knowledge of the succeeding generation of children; b. Knowledge attained by agents with the highest earning capacity (family-1 members) has a spillover effect on all other families (see below). Human capital formation is thus perceived to be a social, as well as a private, process. Knowledge transmission is modeled as an exogenous process in our analysis, assuming that knowledge spillover effects cannot be fully internalized a priori.[5]

The intergenerational interaction is captured by the relationship between H^i_{t+1} and H^i_t in equation (1). The inter-family interaction is defined by the term $(S^i_t)^\gamma$ in equation (1), where $S^i_t \equiv [(\overline{H}^1+H^1_t)/(\overline{H}^i+H^i_t)][N^1_t/N^i_t] \equiv E^i_t P^i_t$. The ratios E^i_t and P^i_t reflect the relative earning capacities and family-group sizes of agents in group 1 relative to i in generation t, and $\gamma<1$ is a spillover-intensity parameter. This specification is designed to capture a continuous social-interaction, or knowledge spillover effect, by which agents with lower earning capacity, or effective knowledge (group i), benefit from interactions with leaders in knowledge (group 1) in various contexts.

The relevance of E^i_t is straightforward: the greater the disparity in knowledge, the greater the potential learning benefit for a member of i from knowledge possessed by 1. P^i_t is a measure of the intensity of social interaction between members of groups 1 and i. The rationale is as follows: we assume that the homogeneous members of group 1 are the exclusive source of knowledge transfer, and that successful transfer of knowledge from a member of group 1 to a member of group i>1 requires **pair-wise interactions** between the two. The real-world scenarios we envisage are the random pairing of agents forming a work team, or sharing a school desk, or a two-seat assignment on a commuter plane, which can lead to intensive knowledge transfer. The odds that the pairing includes a member of group i>1 and another group j≠i is $(N-N^i)/N^i$, where N is the total population. However, this pairing can lead to a productive interaction only if the member from group i meets a member of group 1, the conditional probability of which is given by $N^1/(N-N^i)$. The odds of effective social interaction between members from groups 1 and i is thus given by the product of the odds of encounter between an agent from group i and an agent from another group, adjusted by the conditional probability that the latter agent is from group 1, or $P^i = N^1/N^i$.[6] In the two-group case, this measure becomes simply the odds of encounter between agents from groups 1 and 2 (or the conventional "teacher-student ratio").

Preferences and Motivating Forces: We take parental altruism to be the major force motivating parents' demand for children. However, we also allow for old-age insurance as a complementary motive, as in EL (1991): children are dependent on parents for nurture and educational investments, and old parents can benefit from such investments through informal care provided by adult children. This is our **benchmark case**. While some of the model's basic implications can be derived when parents are driven solely by altruism, we base our analysis on this benchmark case, since it assures the existence of interior solutions for fertility, human capital investments, and thus all inequality measures in all steady states. The pure altruism case is discussed in **Appendix B**.

The utility function of a representative member of family group i at period t is

(2) $U(C^i_{1,t}, C^i_{2,t+1}, C^i_{3,t+1}) = [1/(1-\sigma)][{C^i_{1,t}}^{1-\sigma} - 1] + \delta[1/(1-\sigma)]\{[{C^i_{2,t+1}}^{1-\sigma} - 1] + [{C^i_{3,t+1}}^{1-\sigma} - 1]\}$,

where δ is an intertemporal discount factor, and σ the inverse of the intertemporal elasticity of substitution in consumption. In equation (2), $C^i_{1,t}$ denotes consumption of young adults:

(3) $C^i_{1,t} = (\overline{H}^i + H^i_t)[1 - v^i n^i_t - \theta^i h^i_t n^i_t] - w^i_t H^i_t$.

The control variable, n^i_t represents the number of children per parent, treated as a continuous and certain variable. The endogenous size of group i thus evolves over time as $N^i_{t+1} = N^i_t n^i_t$. The parameter v^i is the fixed cost of quantity of children, and θ^i is the unit cost of financing educational investments per child, which may vary across family types because of capital-market imperfections.

Old-age consumption per parent in equation (2) is given by

(4) $C^i_{2,t+1} = n^i_t w^i_{t+1} H^i_{t+1}$,

Since agents retire at old age, and we initially abstract from savings, old-age consumption depends strictly on material transfers from children. We assume that young parents form implicit contracts with their children that can be enforceable and time consistent (see EL, 1991). By this contract, each adult child transfers to the old parent an amount of support that is proportional to the stock of human capital created by the parent, $w^i_{t+1}H^i_{t+1}$. For simplicity, we first treat the compensation rate, w^i_{t+1}, or **material rate of return** per unit of H^i_{t+1}, as exogenously set by social norms, but we reach all our basic propositions by treating w^i_{t+1} as endogenously determined (see **Appendix C**)[7].

The last term in equation (2),

(5) $C^i_{3,t+1} \equiv B^i(n^i_t)^{\beta}(\overline{H}^i+H^i_{t+1})^{\alpha}$, with $\alpha=1$ and $\beta>1$,

defines the conventional parental altruism function in an OLG context, reflecting psychic rewards parents obtain vicariously from children's full income. The restrictions on α and β are necessary to assure interior solutions for n^i_t and h^i_t (note that a growth equilibrium cannot be sustained if $\alpha>1$). To ensure the concavity of equation (2) we must further restrict $\alpha(1-\sigma)<1$ (or, $\sigma>0$) and $\beta(1-\sigma)<1$.

B. Basic Solutions

The objective function (2) is maximized by choosing the control variables, n^i_t, and h^i_t, subject to (1) and (3)-(5), taking $\{H^i_t, H^1_t, N^i_t, N^1_t, w^i_t, w^i_{t+1}\}$ as given. By substituting the constraints into (2), we derive first-order conditions for n^i_t and h^i_t as follows:

(6) $[C^i_{2,t+1}/ C^i_{1,t}]^{\sigma} \geq \delta R^i_{n,t} \equiv \delta A^i w^i_{t+1}(S^i_t)^{\gamma}(1+\beta M^{i*}_t)/[\theta^i +(v^i/h^i_t)]$, for $n^i_t \geq 0$, and

(7) $[C^i_{2,t+1}/ C^i_{1,t}]^{\sigma} \geq \delta R^i_{h,t} \equiv \delta A^i w^i_{t+1}(S^i_t)^{\gamma}[1+\alpha M^{i*}_t H^i_{t+1}/(\overline{H}^i+H^i_{t+1})]/\theta^i$, for $h^i_t \geq 0$,

where $M^{i*}_t \equiv (C^i_{3,t+1}/ C^i_{2,t+1})^{1-\sigma}$; R^i_n and R^i_h are the rates of return to investments in n and h.

Equations (6) and (7) confirm that in order for interior solutions for n^i_t and h^i_t to exist, i.e., for $R^i_{n,t}$ and $R^i_{h,t}$ to equalize over all development phases, we must restrict $\beta>\alpha$, and $\alpha = 1$. This restriction and the equality of the rates of return $R^i_{n,t} = R^i_{h,t}$ also imply that

$$(8)\ 1 = [B^i(n^i_t)^{\beta-1}/w^i_{t+1}]^{1-\sigma}\,[(\overline{H}^i+H^i_{t+1})/H^i_{t+1}]^{1-\sigma}\,[\beta(\theta^i h^i_t/v^i) - (1+\theta^i h^i_t/v^i)\,H^i_{t+1}/(\overline{H}^i+H^i_{t+1})].$$

An interesting feature of equations (6)-(8) is that ability, A^i, and the financing cost of educational investments, θ^i, exert **opposite** effects on n_t^i and h_t^i, or a "quantity-quality" tradeoff. Indeed, if we rewrite equations (6) and (7) as solutions for n_t^i and H^i_{t+1}, rather than h_t^i, the solutions would depend on the **ratio** $e^i \equiv A^i/\theta^i$, or families' relative "investment efficiencies".

C. Income Inequality Measures

Three income inequality measures become endogenous state variables in our model:

a. $E^i_t \equiv (\overline{H}^1+H^1_t)/(\overline{H}^i+H^i_t)$ is a **family-income inequality** index: the ratio of the (full) income of an individual family in family-group 1 to that of a corresponding family in family-group i. An inequality measure directly related to E^i_t in our model is inequality in attained human capital stocks, H^1_t/H^i_t, which may be captured roughly by the standard deviation of schooling attainments.

b. $S^i_t \equiv [(\overline{H}^1+H^1_t)/(\overline{H}^i+H^i_t)][N^1_t/N^i_t] \equiv E^i_t P^i_t$ is our **income-group** (or **income-bracket**) **inequality** index – a product of relative income levels and group sizes of family-group 1 relative to i>1– which is also a component of the social interaction term in equation (1). It measures the proportion of **aggregate income** held by members of the top income bracket (above a given dollar value), relative to lower brackets. Note that $P^i_t \equiv N^1_t/N^i_t$ is a related endogenous distributional measure – a **income-group-size inequality index**. It measures the proportion of families in the top income bracket relative to those in lower brackets, or the relative distribution of families across income groups. The latter is not independent of S^i_t and E^i_t since, by definition, $P^i_t \equiv S^i_t/E^i_t$.

c. The **Gini coefficient**, $G_t \equiv \sum_{j=1}^{I}\sum_{k=j+1}^{I}[(1/P^k_t)(1/S^j_t) - (1/P^j_t)(1/S^k_t)] \,/\, [\sum_{j=1}^{I}(1/P^j_t)\ \sum_{j=1}^{I}(1/S^j_t)]$, turns out to be a non-linear function of S^i_t and P^i_t. In the two-family case, the Gini coefficient becomes $G_t \equiv (S^i_t - P^i_t)/[(1+ S^i_t)(1+ P^i_t)]$, which is increasing in S^i_t, but decreasing in P^i_t.

III. EQUILIBRIUM REGIMES AND COMPARATIVE DYNAMICS

Equations (6) and (7) represent complex second-order simultaneous difference equations, and generally no explicit solutions exist for the model's basic endogenous variables, n^i_t, h^i_t and S^i_t. Stable equilibrium solutions can be obtained, however, through simulations satisfying our parametric restrictions along with the system's second-order optimality conditions. The simulations indicate that two locally stable steady states exist, corresponding to different parameter values: stagnant (s) and perpetual growth (g) equilibrium. [8] The transitional development phase connecting the two is supported by the same parameter set that sustains the perpetual growth steady state.

In deriving these alternative development phases, it is necessary to impose some restrictions on the distribution of specific parameters across family groups. Given our assumed uniformity of preferences and external production parameters, $V \equiv \{B, \alpha\ (=1), \beta, \gamma, \delta, \text{and } \sigma\}$, we can show that only initial endowments ($\overline{H}^i$), abilities (A^i), and unit investment costs (θ^i) can be allowed to vary across families. We shall henceforth refer to this as our **heterogeneity restriction**. In particular, if the vector V is identical in all families, a sufficient condition for stable stagnant and growth equilibrium steady states to exist is that the shares of earnings spent on raising children, v^i, and supporting old parents, w^i, must be **identical** as well.[9]

Proposition 1. Both fertility rates and the marginal rates of change of human capital formation of different family groups must converge in **any** stable equilibrium steady state. Formally, we expect:

(9) $n^1_t = n^i_t$, and $a^1_t \equiv (dH^1_{t+1}/dH^1_t) = A^1h^1_t = a^i_t \equiv (dH^i_{t+1}/dH^i_t) = A^ih^i_t(S^i_t)^\gamma$ for all i>1.

Proof: In any stable steady state with heterogeneous families, the distribution of families by income class $P^i_t \equiv N^1_t/N^i_t$ must be stable over time, which also requires that fertility rates equalize across families. Suppose there is an exogenous shock which raises the initial fertility level in family 1, n^1_t, above that of family i, n^i_t. This will increase P^i_t and the social interaction term in equation (1), S^i_t, which will raise the rates of return to investment in both quantity and quality of children. The fertility rate in family-group i will rise relative to that of family 1 (which is unaffected by S^i_t), subsequently depressing P^i_t and S^i_t. The imbalance would continue until fertility rates equalize. This result is consistent with optimal behavior by families since in a stable equilibrium steady state, the impact of a lower income level is offset by a proportionately lower shadow price of fertility.[10]

The proof for why the marginal rates of change of human capital formation, a^i_t, must equalize is similar. Suppose that the families' equilibrium human capital attainment ratios are stable over time, or $H^1_{t+1}/H^1_t = H^i_{t+1}/H^i_t$. If an exogenous shock raises agent 1's marginal rate of change of human capital above that of i, or $a^1_t > a^i_t$, our income inequality measures, both E^i_t and S^i_t, would rise. A higher social interaction level S^i_t raises the rate of return to human capital investment for family i, thus a^i_t, while a^1_t is independent of S^i_t. The adjustments would persist until the marginal rates of change equalize. In a growth steady state, this condition implies equal growth rates of human capital and income across all families.

A. Stagnant Equilibrium (SE) Steady State.

As we document in section C below, if the parameters affecting the rate of return on human capital, A^i/θ^i, v^i, w^i, are sufficiently low, the only stable steady state is a stagnant equilibrium (SE). The conditions under which this equilibrium exists require that in the neighborhood of the SE: a. the evolution path of H^i_{t+1} as a function of H^i_t intersects the 45% degree line from above, i.e., the slope $a^i_t(s) \equiv dH^i_{t+1}/dH^i_t$ is less than 1; b. The families' fertility rates do not increase with family-group sizes, or $dn^i_t/dN^i_t \leq 0$, so that $P^i_t \equiv N^1_t/N^i_t$ converges to a steady state value, $P^i(s)$. Our simulations also consistently yield a **single** stable SE solution under these conditions.[11] In this steady state, we obtain a strong inference concerning the determinants of family-income inequality:

Proposition 2. In a stable stagnant-equilibrium steady state, family-income inequality, $E^i(s)$, and families' relative human capital attainments equal their relative inherited income endowments:

(10) $E^i(s) = [H^1_t/H^i_t](s) = \overline{H}^1/\overline{H}^i$, all i.

Proof: Equation (10) is obtained utilizing equation (1), the stagnancy of human capital attainments over time, and proposition 1, requiring that marginal rates of change of human capital equalize across families, or $a^1=a^i$. The solution is intriguing: status differences are the sole factor determining family income inequality in a SE. It is also stable: suppose we start from a stable SE. If a parameter shock raises H^1_t/H^i_t above $\overline{H}^1/\overline{H}^i$, then E^i_t exceeds $\overline{H}^1/\overline{H}^i$ and S^i_t rises. This raises a^i_t over a^1_t, which lowers H^1_t/H^i_t until it becomes equal to $\overline{H}^1/\overline{H}^i$.

Given our heterogeneity restriction, and assuming that a stagnant-equilibrium steady state exists, we can also solve for the equilibrium income-group inequality index, $S^i(s)$. By Proposition 1 and using equations (1), (6), (7), and (10) we obtain:

(11) $R^1_h=R^i_h=R^1_n=R^i_n$ and $\theta^1 h^1 = \theta^i h^i$.

Combining Proposition 1 and eq. (11), we can derive explicitly the equilibrium $S^i(s)$ measure:[12]

(12) $S^i(s) \equiv E^i(s)\, P^i(s) = [(A^1/\theta^1)/(A^i/\theta^i)]^{(1/\gamma)} \equiv (e^1/e^i)^{(1/\gamma)}$.

Proposition 3. Given our heterogeneity restriction, in a SE steady state the shares of full income devoted to human capital investments, $\theta^i h^i(s)$, and the rates of return on quantity and quality of children are equalized across all family groups, while income-group inequality, $S^i(s) \equiv E^i(s)P^i(s)$

depends strictly on the relative "investment efficiencies" of family 1 relative to i, $(e^1/e^i)^{(1/\gamma)}$. Unlike $S^i(s)$, however, the size distribution of families across income brackets $P^i(s)$ and the Gini coefficient G(s) depend on both relative family endowments and investment efficiencies.

Proposition 3 has an intuitive interpretation: By equation (9), the only endogenous parameter that can adjust to satisfy proposition 1's requirement of equal marginal rates of change of human capital across family groups, $A^1h^1=A^ih^i[S^i(s)]^\gamma$, is the social interaction term. Adjustments in investment behavior are strictly a function of relative investment efficiencies (equations 6-8). Therefore, adjustments in $S^i(s)$ must be a function of relative investment efficiencies as well.

Propositions 2 and 3 offer a set of comparative dynamic implications in the stagnant steady state. By Proposition 2, family income inequality, $E^i(s)$, is strictly a function of families' relative endowments. Any changes in relative inequality across income brackets, $S^i(s)\equiv E^i_t(s)P^i_t(s)$, are therefore made via adjustments in relative family group sizes, $P^i(s)=[N^1/N^i](s)$. For example, an increase in the intensity of knowledge spillover, γ, leaves family choices intact, but lowers income-group inequality, $S^i(s)$, thus $P^i(s)$. Changes in the common values of other parameters can affect fertility, $n^i(s)$, and human capital investment, $h^i(s)$, but not any of our income inequality measures. Higher fixed cost of fertility ($v^1=v^i$) raises h^i and lowers n^i in all families. Stronger altruistic preferences ($B^1=B^i$), in contrast, yield just the opposite effects. (See Table 1 part 1.)

Is income inequality related systematically to income **levels** in a stagnant equilibrium? Note that although income levels are stagnant over time, they can vary with parameter changes. For example, a skill-biased technological improvement raising family 1's relative investment efficiency, e^1, initially raises the rate of return to h^i over n^i, thus ultimately $H^i(s)$ and income levels in all families. By proposition 2, however, family-income inequality, $E^i(s)$, remains unchanged, while by proposition 3, income-group inequality, $S^i(s)$ rises because the wealth effect generated by a higher e^1 initially increases group 1's relative fertility, thus ultimately its population share $P^i(s) \equiv [N^1/N^i](s)$. The impact on Gini is generally **ambiguous**, since G(s) rises with $S^i(s)$, but falls with $P^i(s)$.[13] In our simulations, however, the latter effect dominates (see Table 1 part 1). In the SE, a specific parameter change may thus change different inequality measures in different directions.[14]

B. Growth Equilibrium (GE) Steady State.

Here the state variable H^i_t grows without bound with constant, long-run values of $n^i(g)$, $h^i(g)$, and all inequality measures. Since the role of the income-producing endowments vanishes, proposition 1 implies that the long-run growth rate of human capital in all groups converges on its marginal value in family group 1, $\lim_{t\to\infty}(H^i_{t+1}/H^i_t) \equiv a^1(g)=A^1h^1(g) = a^i(g) =A^ih^i(g)S^i(g)^\gamma$. Local stability is assured if the slope of the evolution path of H^i_{t+1} as a function of H^i_t, $a^i(g) \equiv dH^i_{t+1}/dH^i_t$, **exceeds** 1 and the fertility rates cannot be increasing with group sizes, or $dn^i_t/dN^i_t \leq 0$.[15]

Proposition 4. Proposition 3 remains valid at the **growth equilibrium** steady state as well. Moreover, if the distribution of the heterogeneous parameters A^i and θ^i remains the same in the SE and GE, income-group inequality S^i would converge on the **same level** in both steady states:

(13) $S^i(g) = [(A^1/\theta^1)/(A^i/\theta^i)]^{(1/\gamma)} \equiv (e^1/e^i)^{(1/\gamma)} = S^i(s)$, all i.

The proof is the same as for Proposition 3 (fn. 12). The comparative-dynamic implications of equation (13) are also similar to those of equation (12): $S^i(g)$ rises with the relative investment efficiency (e^1/e^i) and falls with the spillover coefficient γ, as is the case for the SE steady state.

Unlike the stagnant steady-state case, however, where family-income inequality $E^i(s)$ was determined **strictly** by the relative income producing endowments across families (see eq. 10), the relative influence of endowments vanishes under persistent growth. The comparative values of E^i in the GE vs. SE steady states thus depend on inter-family differences in investment efficiencies, as well as on the factors determining the **evolution** of E^i_t along the development phase linking the two steady states. The same holds for the income-group-size inequality index, P^i_t.

Comparative-dynamics effects of parameter shocks on the family-income ($E^i(g)$) and income-group-size inequality index ($P^i(g)$) inequality indices are thus ambiguous. A **skill-biased** technological advance favoring the leading family (a rise in e^1/e^i), which by proposition 4 unambiguously raises $S^i(g)\equiv E^i(g)P^i(g)$, must raise either $E^i(g)$, or $P^i(g)$, or both. $E^i(g)$ necessarily rises if the upward shift in A^1 raises the growth rate of human capital in family 1, $A^1h^1_t$, above that in family i **all along** the transitional dynamics path. This is the case if the utility function is logarithmic (σ=1), since in this case, changes in A^1 or γ have no effect on h^i or n^i, and hence on $P^i(g)$ (see footnote 15). If $\sigma\neq1$, fertility may rise while h falls with a rise in A^1, essentially because such a shock does not alter the relative rate of return on n and h (unlike the stagnant equilibrium case), while the pure wealth effect generated by a higher e^1 favors fertility. The effect on the Gini coefficient is ambiguous if both $S^i(g)$ and $P^i(g)$ increase as a result, since G(g) rises with the former and falls with the latter. However, in all our simulations in **Table 1 part 2**, G(g) moves in tandem with all other income inequality measures.

These results appear to be consistent with the US experience following the "Information Technology revolution": empirical studies have shown that income inequality rose in the 1980s (see e.g., Katz and Murphy, 1992). Moreover, as Census data indicate, fertility levels have actually reversed a historic downward trend since the baby boom and started growing from 1.74 in 1977 to a peak of 2.08 in 1990, remaining steady at about 2.02 thereafter, and the coefficient of variation of fertility rose from 0.619 in 1983 to 0.710 in 1994 and has remained stable since then.

Table 1, part 2 also indicates that common parameters such as v and B, which do not affect any income inequality measure, affect the growth rate in opposite directions: a rise in v lowers $n^i(g)$ and raises $h^i(g)$, thus the growth rate, while a rise in B lowers both. A skill-biased technical change, in contrast, raises all income inequality measures and the growth rate as well. The association between income growth rate and income inequality thus depends largely on the parameter changes responsible for their co-movements.

C. Takeoff Triggers.

Whether the economy is in a stagnant or growth equilibrium depends on the magnitude of the model's basic parameters. An upward shock in A^i/θ^i, v, or w (treated as exogenous) raises the rates of return to both quantity and quality of children, but also the latter over the former, $R^i_{h,t}/R^i_{n,t}$ (see equations 6 and 7). Indeed, a sufficient shock, even one affecting just group 1, can generate a takeoff for all groups. Our simulations produce another key feature of economic development – the "demographic transition"– whereby fertility generally declines while investment in human capital increases as a result of a sufficient upward shift in any of our takeoff triggers (see **Table 1 part 3**). In the logarithmic utility case, this can be shown analytically as well (see fns 11 and 15).

IV. INEQUALITY PATHS OVER THE TRANSITIONAL DEVELOPMENT PHASE

A. Paths of Income Inequality Measures

The preceding analysis indicates that the behavior of inequalities over the development phase partly depends on the type of shock that produces a takeoff. An equally important issue is how fast any given shock reaches different family groups: a skill-biased technological advance, e.g., is likely to first reach the group with the highest investment efficiency, or affect it proportionally more than other groups. While this group need not necessarily be the one with the highest income – this depends on the correlation between ability and initial endowments across family groups – a positive correlation is likely, as conjectured by Becker (1967). To contain the possible scenarios we focus on three that are neither exhaustive nor necessarily of equal empirical plausibility:

a. Synchronous and uniform shocks: These shocks affect all takeoff-triggering parameters (A/θ, v, w) simultaneously and by the same proportion. This case can be dubbed as "the separating equilibrium path"; we can show that over the transitional phase:

(14) $S^i_t = S^i(s) = S^i(g) = [(A^1/\theta^1)/(A^i/\theta^i)]^{(1/\gamma)}$, and $E^i_t = E^i(s) = E^i(g) = \overline{H}^1/\overline{H}^i$.

Put differently, our basic earnings inequality measures chart a horizontal path all along the development process. This is because a uniform proportional increase in a takeoff-triggering parameter affects all optimality conditions symmetrically, leaving constant the spillover effect. Since the Gini coefficient is a function of S^i and P^i, it also exhibits a flat transition path.

b. Shocks favorable to family 1: Such a shock affects family 1 either proportionally more than other families, or ahead of other families. An example would be a takeoff-triggering technical advance that enhances more the skill of family 1(A^1), or complements all families proportionally, but is first integrated by family 1. We implicitly assume a positive correlation between income-generating endowments and efficiency at human capital investments, or COV($\overline{H}^i$, A^i/θ^i) > 0, so the higher-income family 1 is a leading group at both the stagnant and growth steady states.

If family 1 is affected ahead of other families, the transitional development phase would be characterized by the co-existence of family groups in different stages of transition: Family 1 would initially become a **"growth family"** while other families remain "stagnant families". But the persistent growth in family 1's income ultimately produces a takeoff for all, and by proposition 1, all will ultimately grow at an equal rate. The time paths of all income inequality measures (S^i, E^i, and G) will exhibit an **inverted-U shape**, consistent with the "Kuznets hypothesis".

Whether the income inequality **level** at the growth equilibrium is higher or lower than at the stagnant equilibrium depends on whether the shock ultimately affects all families uniformly, i.e., equi-proportionally. An ultimate uniform shock does not affect the GE income-group inequality level $S^i \equiv E^i P^i$ by equation (13), but it raises the GE family-income inequality level, $E^i(g)$, because of the demographic transition triggered by the jump in A, which lowers the fertility level in group 1 ahead of group i and thus the relative size of group 1, $P^i(g)$ (see **Figure 1**). If the shock raises A^i proportionally more for family 1, income inequality would then be **monotonically increasing** over the development phase for **all** our three income inequality measures.

c. Shocks favorable to family i: A family-i friendly shock could occur, e.g., when a less segmented capital market lowers the education financing cost to all families, but especially to family i, thus lowering (e^1/e^i), or when the shock first benefits family i members, who could not initially finance private schooling. In this case, the takeoff-triggering shocks will produce transition paths just opposite to those in case b. The time paths of all inequality measures will assume a **U shape** if family i experiences a takeoff shock ahead of family 1 (see **Figure 2**). Whether the inequality **level** rises or falls at the GE, relative to the SE steady state depends on whether the non-synchronized shock ultimately becomes equi-proportional, in which case the income-group inequality, S^i, is constant and $P^i(g)$ rises, so family-income inequality, $E^i(g)$, falls, or if investment efficiency rises proportionally more for family i, in which case the income inequality level is **monotonously decreasing**.[16]

Our simulations of cases b and c also reveal opposite associations between income **growth** rate and income **inequality** over the transitional development phase. In case b, income inequality and per-capita income growth rate are positively associated, as Forbes (2000) finds, while in case c they are negatively associated at an early stage of the transition, but become positively associated at a more advanced stage, which is what Barro (2000) finds. Our analysis thus shows that the dynamic association between income growth and income inequality can vary by the specific takeoff triggers, or at different stages of the transitional development phase.

Regardless of the way a takeoff-generating shock affects different families, a general implication of our model is that the shape of the family-income inequality path over the transition $E^i_t \equiv (\overline{H}^1+H^1_t)/(\overline{H}^i+H^i_t)$, would always be consistent with that of human capital attainments, H^1_t/H^i_t, regardless of the shape of the paths. Our simulations indicate that this applies to our other measures of income inequality as well.

B. Paths of inequality in fertility and human capital investment.

Since by propositions 1, 3, and 4 optimal fertility and the shares of income spent on educating each child are equal for all families at both the SE and the GE steady states, while they generally deviate across families during the transitional phase connecting the two, we have:

Proposition 5. Except in the "separating equilibrium" case, the transitional development path of inequality in completed fertility n will exhibit an **inverted-U shape**, but tend toward equality in the two steady states framing the transitional phase. The same applies to the transitional development path of the educational investment cost shares, θh. (See Figures 1d and 2c.) In the separating equilibrium case, the inequalities in n and h assume a flat time path.

If income inequality measures assume an inverted-U shape, as in case b of the preceding section, family 1's fertility level is lower than that in family i over the transitional development phase (see Figure 1a). This association between fertility rankings and income inequality is consistent with the findings in Kremer and Chen (2002) and De la Croix and Doepke (2003). In contrast, when income inequality assumes a U shape, as in case c, family 1's fertility exceeds that of family i during the transitional phase. In our GE framework, however, such associations do not indicate causality, nor can they be persistent, since fertility differences vanish in any stable steady state.

V. EMPIRICAL ANALYSIS

A. Basic tests

We test empirically two basic implications of the model: a. By Proposition 5, we expect **fertility inequality** to display an **inverted-U shape with flat tails** over the development phase (except in the "separating equilibrium" case); b. We similarly expect inequality in **human capital investments** to exhibit an inverted-U-shaped path over the transitional development phase. No reliable data on investment **flows** are available internationally. However, using schooling levels to approximate human capital attainments, we predict the shapes of **schooling** and **family-income inequality** paths to be similar over the transition phase.[17] By our simulation analysis, this expectation applies to all our income-inequality measures as well.

B. Data and Variables Used

a. Completed fertility. Distributional data on the number of surviving children per woman are available from the World Fertility Surveys (WFS) and their successor — the Demographic and Health Surveys (DHS). The sample we construct is based on 72 surveys of 29 developing countries in various years between 1974 and 2000. From the individual-level micro data in each survey, we derive the distribution of surviving children of women age 40 an over. We make this restriction to insure that our measures relate to women who completed childbearing. We then use the **standard deviation** of the distribution of surviving children per woman age 40 and over [SD-FERT] as our fertility inequality measure. But since the standard deviation is subject to a secular drift, we also enter the average **level** of completed fertility as a control variable, [AV-FERT].

b. Human capital. The source of educational attainments data (schooling years in the population age 15 and over) is Barro and Lee (2000).[18] We use the average number of years of schooling in the population age 15 and over as a proxy for human capital stock. As a measure of inequality in educational attainments we use the standard deviation of the distribution of schooling years [SD-SCHYR] in the population age 15 and over. As in the fertility inequality regressions, we also add the mean schooling years as a control variable [AV-SCHYR].

c. Income inequality. The data are taken from Dollar and Kraay (2001). These data cover 86 countries over the period 1950-1998. No data are available to compute our income-group inequality measure, S^i. We proxy our family-income inequality measure, E^i, however, by an inter-quintile income inequality ratio [QUINT] (each 'quintile' representing, by definition, an equal number of households), and we measure G by the actually measured Gini coefficient [GINI]. To be consistent with our model, we use only observations that are calculated from **household income** data, excluding observations based on personal income and expenditure data.

d. Regressors. We use real GDP level [RGDP], as reported in Heston, Summers, and Aten [HSA] (2001) to account for the economy's level of development. As a robustness check, we also enter the time trend itself as a control variable. We use the government share of GDP [GOV] and the share of government educational expenses in GDP [GED] to account for the role of government spending in affecting our distributional variables. These variables are taken from HSA and UNESCO, respectively. Summary statistics for all variables are reported in Appendix D.

C. Regression models

Our basic specification is an OLS regression in which our inequality measures and the regressors are entered in natural form, but RGDP is entered in **cubic** or **higher-order** polynomial form. This is because we predict the **flattening** of the income inequality path as the economy converges on a growth steady state. Using OLS is consistent with our model, since we expect the relation between our inequality variables and the development level to be associative, not causal.

To examine the robustness of our results, we also test several modifications. The first uses

OLS with fixed country (models 2-4), and year (model 5) effects. Models 2-4 thus capture "within-countries" variability in the regressors, whereas model 5 captures "within-calendar-year" variability. In models 3-5, we also test the effect of the government's share of GDP, GOV. In model 4, we include a time trend variable T to account for missing trended controls.

In the fertility regressions of Table 2, we employ country-specific **random-effects,** instead of fixed-effects, models to increase the regressions' degrees of freedom, because the number of observations per country is small (2.6 per country). The results from the fixed-effects specification are similar qualitatively, but the regression coefficients have larger standard errors.

To test for serially correlated errors, we have applied an AR(1) serial correlation test to model 3 of each table. The Cochran-Orcutt test rejects the null hypothesis in all cases.

D. Results

The fertility results are reported in Table 2. Regression model 1 estimates an inverted-U-shaped association between fertility inequality and real income, with inequality peaking at an RGDP level of $3,538 (note that our sample is dominated by developing countries, so 67% of the observations lie below this real GDP level and 33% above it). The estimated association is depicted in **Figure 3a**. The shape remains virtually the same when we apply random-effects regressions – with or without calendar-year dummy variables. As for other regressors, the standard deviation of the fertility distribution is monotonically related to the distribution's mean, as one would expect for any distribution. GOV - a proxy for the average income tax - has a negative and significant effect only in the Gini regressions. Public education expenditures [GED] generally have the same effect as GOV, but because it entails a very large reduction in sample size, we do not report these regressions. The time-trend regressor, introduced to account for missing trended factors, is inversely related to fertility inequality, but is directly related to educational attainments inequality.

Table 3 reports the results concerning inequality in educational attainments. Regression results indicate an inverted-U-shaped association between income and educational-attainment inequality, as depicted in **Figure 3b**. Mean schooling expectedly raises the SD of schooling.

Tables 4 and 5 estimate the shape of the income inequality path for the Gini coefficient and QUINT (as a proxy for E^i_t), respectively. All model specifications indicate an inverted-U-shaped association between income inequality and income level. In model 6 of both tables we derive this association based on the subset of countries for which both income and educational attainments data are available. We do so because our model and simulation results imply that family-income inequality (E^i_t) and human capital distribution paths would exhibit an increasingly similar shape as the economy converges on a GE steady state. The results of model 6 are depicted in **Figures 3c** and **3d**, which are found to be surprisingly similar to Figure 3b depicting the educational attainments inequality path.[19] This lends support to our human capital approach to income distribution. While income inequality at the higher-income range is falling, we cannot determine from this evidence if income inequality is higher or lower at the growth vs. stagnant steady states, since all countries in our sample may have already reached an advanced transition stage toward a GE steady state.

Tables 3-5 have special significance from our model's perspective: since the estimated associations between educational attainments and income level, and between our income inequality measures and income level take on an inverted-U shape, the results militate in favor of the Kuznets hypothesis. Note, however, that these results cannot be taken to support the Kuznets hypothesis as a general "law": our analysis indicates that the observed association can be affected by the specific **composition** of countries in our sample, in terms of the development stage they have achieved, as well as by the specific takeoff triggers operating in different countries.

Our results concerning the dynamic behavior of income inequality can be compared to those of Deininger and Squire [DS] (1998). Although DS employ the same data, they use a different fixed-effects regression format where RGDP is entered via two variables, RGDP and 1/RGDP. When we add a cubic or higher-order form of RGDP to the DS specification, however, the plotted relationships between GINI or QUINT and RGDP exhibit inverted-U shapes in this specification as well, similar to those in Figures 3c and 3d.

VI. CONCLUDING REMARKS

The main message of this paper is that income distribution in the population is determined fundamentally by the corresponding distribution of human capital attainments, not just under static conditions, as in Becker's seminal 1967 paper, but under dynamic conditions as well. In this context, propositions concerning the behavior of income inequality over a transitional development period – what Kuznetz had focused on – should be explained by the dynamic behavior of inequality in educational attainments. Our paper's main insight is that the behavior of the educational inequality path, in turn, is linked to that of its two main determinants: investments by the parents' generation in the quantity (n^i) and quality (h^i) of their offspring. This linkage enables us to derive theoretical income inequality paths over the transitional development period, as well as inferences about the income inequality levels in the two steady states which frame it.

Although our analysis is based on a deterministic model of heterogeneous families differing in abilities, investment efficiencies, and inherited endowments, it allows for social mobility as well, as would be the case if leadership in human capital formation switches over the development phase from the group with initially highest human capital attainments to a group with an initially lower one. We can also extend our model to allow for stochastic variations in ability within groups as a major source of intra-group heterogeneities. This extension provides additional dynamic implications about social mobility, but does not alter the main propositions of our paper (see footnote 4). Furthermore, although our benchmark model relates to inequality in earning capacity, the propositions we derive can be shown to apply to total family income as well (see Appendix A).

A central implication of our analysis is that regardless of the dynamic pattern of any of our income inequality measures, which remain positive over time, the inequalities in both fertility and educational investments are expected to have an inverted-U shape over the transitional development phase with attenuated tails. This prediction is borne out by our empirical analysis in section V, based on data from developing and developed countries. It is also consistent with historical evidence indicating that relative variances of fertility in most Western European countries exhibit an inverted U-shaped path between the mid-19th century and 1970, and were quite small in the pre-demographic transition phase, although this evidence relates to variations in fertility levels across provinces, rather than families (see Coale and Treadway, 1986).

Concerning the dynamic association between income growth and income inequality, our model offers several insights. First, no general "law" can be derived, since the association reflects the impact of underlying parameter changes that trigger co-movements in both income level and income inequality. We can thus account, in principle, for empirical studies producing conflicting shapes of income inequality paths over a transitional development period. We show, first, that the observed association can go in similar or opposite directions depending on the way specific parameter changes affect different family groups. Second, the association depends on whether the economy is in a stagnant- or growth-equilibrium steady state, or in a transitional development phase, which makes the results vulnerable to the specific mix of countries in the sample. Third, the association partly depends on the inequality measure used. We derive three such measures as endogenous variables: family-income inequality (E^i_t), income-group inequality (S^i_t=E^i_t P^i_t), which also depends on an income-group-size inequality measure (P^i_t), and the Gini coefficient (G_t). We show that G_t is a non-linear function of the latter two: it is increasing in S^i_t but decreasing in P^i_t. Our model offers some strong predictions concerning the dynamic behavior of these measures.

For example, under an assumed homogeneity of preferences and a stable distribution of abilities, or investment efficiencies, we expect the relative distribution of income across income groups, or brackets, to converge on the same level in stagnant and growth steady states, $S^i(s)$= $S^i(g)$. This specific measure of relative income inequality, which may be approximated empirically by the relative shares of aggregate income going to different income brackets, may thus exhibit a high degree of stability over the long haul. We expect family-income inequality in a stagnant steady state, $E^i(s)$, to be strictly a function of inequality in family-specific endowments such as political or legal status, but show that the role of these inherited endowments gradually vanishes as the economy converges on a growth-equilibrium steady state. No clear-cut predictions can be made, however, about the shape of the family-income inequality path over the transition phase: this measure is affected by the specific takeoff-triggering parameter and the way it impacts different family groups. An inverted-U-shaped family inequality path is likely to emerge as a result of a uniform skill-biased technological advance that first reaches the top (highest skilled) family group, in which case the family-income inequality level would be **higher** at the growth steady state relative to the stagnant equilibrium steady state, which is the approximate shape produced by our empirical investigation in section V. An inverted-U-shaped family-income inequality with inequality turning **lower** at advanced vs. initial development phases, in contrast, can emerge as a result of reduced capital market segmentation, first taken advantage of by more knowledgeable, higher-income families, but which lowers especially the financing-cost disadvantage of lower-income families.

Our analysis can rationalize dynamic changes in income inequality not just over the transitional development period, but within equilibrium growth regimes as well. For example, the information technology revolution experienced largely during the 1980s is predicted by our comparative dynamic analysis to raise the steady-state level of family income inequality, but also the steady state level of fertility. This, indeed, is what the US data show in the 1980s.

A central implication of our model is that the dynamic path of family-income inequality, regardless of its shape, should mirror that of educational attainments, all having flat tails. This is what we find empirically. Our empirically estimated Kuznets-like paths of inequalities in income and schooling attainments may not be general, as we argue theoretically, but the patterns are consistent with each other. This lends support to our thesis that human capital is both the engine of income growth and the main determinant of income distribution paths over the development process.

Appendix

A. Although our benchmark model abstracts from capital markets, we can incorporate returns on savings as an outcome of "home production", whereby the old parents' human capital attainments is an input, and the yield is subject to diminishing returns. This is a natural assumption in the context of our closed-economy framework. The extension allows us to recognize inequalities in labor earnings as well as in total income, incorporating both earnings and property income.

Formally, total savings is defined by $K_t \equiv (\overline{H}^i + H^i_t)s^i_t$, where s^i_t is the fraction of productive capacity saved at adulthood, and K_t is assumed to fully depreciate within one generation. Income from savings is generated when old parents combine their accumulated assets, K_t, with their human capital inputs via the production function, $F = D(\overline{H}^i + H^i_t)^{1-\kappa}[(\overline{H}^i + H^i_t)s^i_t]^\kappa$, $0<\kappa<1$. The consumption flows at adulthood and old age are thus given by

(3′) $C^i_{1,t} = (\overline{H}^i + H^i_t)[1 - v^i n^i_t - \theta^i h^i_t n^i_t - s^i_t] - w^i_t H^i_t$,

(4′) $C^i_{2,t+1} = n^i_t w^i_{t+1} H^i_{t+1} + D(\overline{H}^i + H^i_t)^{1-\kappa}[(\overline{H}^i + H^i_t)s^i_t]^\kappa$.

We can now distinguish income inequality from earnings inequality. The measures of the pooled income of a family head – earnings as well as property income from savings – can be defined parallel to our earnings-inequality measures in section II.C. For example, TS^i_t below corresponds to the ratio of **total** income-group inequality (wage earnings of adult parents plus non-wage income of old parents) of group 1 relative to group i, and the same holds for family income inequality, TE^i_t, and the Gini coefficient, TG^i_t (in the 2-family case):

$TS^i_t \equiv [N^1_t(\overline{H}^1 + H^1_t) + N^1_{t-1} D(\overline{H}^1 + H^1_{t-1})(s^1_{t-1})^\kappa] / [N^i_t(\overline{H}^i + H^i_t) + N^i_{t-1} D(\overline{H}^i + H^i_{t-1})(s^i_{t-1})^\kappa]$,

$TE^i_t \equiv TS^i_t / TP^i_t$; $TP^i_t \equiv [(N^1_t + N^1_{t-1})/(N^i_t + N^i_{t-1})]$, and

$TG^i_t \equiv [TS^i_t - (N^1_t + N^1_{t-1})/(N^i_t + N^i_{t-1})] / (1 + TS^i_t) / [1 + (N^1_t + N^1_{t-1})/(N^i_t + N^i_{t-1})]$.

Under our heterogeneity restriction, we can show that optimal savings (s^i), fertility (n^i), human capital investment costs per child ($\theta^i h^i$), and the rates of returns to these control variables are identical for all family groups in any steady state, since the first-order optimality conditions governing these control variables become identical for all family groups in all stable steady states. Our total income inequality measures are therefore identical to the corresponding earnings-inequality measure at both the stagnant- and growth-equilibrium steady states. Moreover, we can show that the relative inequality in earnings, and hence in total income in this extended model is the same as that derived in our benchmark model sans savings, as given by equations (10), (12) and (13). All of the propositions in sections III and IV are also maintained in this extended model, as are the qualitative results of the comparative dynamics reported in Table 1 for both the SE and GE steady states. The time paths of the inequality measures considered in section IV are also shown to take the same pattern as in the model without savings.

Over the transitional development phase, however, the savings rate may differ across families. For example, when a takeoff occurs as a result of a skilled-bias technological advance reaching initially the higher-income family group 1, our income inequality measures assume an inverted-U shape, and the savings rate of the higher-income family 1 initially falls below that of the other (stagnant) families. In the following stage, however, as family groups i>1 experience a takeoff because of the social-interaction effects coming from family-group 1, their savings rates fall below that of family 1. The aggregate savings rate then starts rising while income inequality is increasing. The resulting positive association between income inequality and the aggregate savings rate does not indicate causality, however (as in Keynes, 1920, or Kaldor, 1957). It eventually reverses in this example, as income inequality starts falling, and it vanishes as all savings rates converge on equality in the GE steady state.

What would be the effect of changes in D or κ on our income inequality measures? As long as these changes are common to all families, they will affect only the **composition** of family income, but not the relative, total income inequality measures, as our simulations confirm.

B. Given that parents' demand for children is motivated strictly by altruism, as specified in equation (5), with no material incentive (i.e., w=0), we can show that under a stable stagnant equilibrium steady state, the first order optimality conditions for fertility hold across all family groups only when none invests in human capital formation, i.e., if $h^1(s)=h^i(s)=0$. This condition can be shown to guarantee that both the rates of return on fertility, and the equilibrium fertility rates would be equalized across all family groups, i.e., $n^1(s)=n^i(s)$. Equality of fertility rates is necessary to assure the existence of any steady state equilibrium by proposition 1, since otherwise one family group becomes dominant in the population, and all inequalities vanish. Note that if we adopt the altruism functions in EL (1991) instead of the one used in this paper, this could lead to interior solutions in both $n^i(s)$ and $h^i(s)$ in the stagnant steady state, but in this case, increases in investment efficiencies cannot produce a demographic transition in fertility.

Since there is no human capital accumulation in this stagnant equilibrium, or $H^i(s)=0$, the family-income inequality ratio is automatically set by the income-generating endowment ratio, $E^i(s)= \overline{H}^1/\overline{H}^i$, as is the case in our benchmark model. But since there is no interior solution for $h^i(s)$, the equilibrium value of the social interaction term, $S^i(s)\equiv E^i(s)P^i(s)$, cannot be pinned down, as is the case under our benchmark model: although the rates of growth of the various population groups are equalized, the population ratio $P^i(s)=N^1(s)/N^i(s)$ depends entirely on the arbitrary values one would assign to the initial values of the population groups. Our income-group inequality measure and the Gini coefficient, $S^i(s)$ and $G(s)$, are thus indeterminate in the stagnant equilibrium steady state. As a result, we also cannot pin down the time paths of S^i_t, P^i_t and G_t during the transitional development phase, and their comparison with their counterparts under a growth equilibrium steady state.

This corner solution in $h^i=0$ at the stagnant state can always be avoided, if we allow for any positive old-age support rate w>0. This is because the marginal return to human capital investment becomes infinite as h^i approaches zero. The pure altruism case can be applied, however, at the growth steady state, and its behavioral implications are qualitatively the same as those we derive for our benchmark case.

C. In this appendix, we treat the old-age support rate, as an endogenous variable, rather than an exogenous constant. We follow EL (1991) in analyzing parents' choice of w^i_{t+1} as a time consistent, principal-agent decision. Parents select values of w^i_{t+1} that maximize equation (2) for children, taking as given the children's optimal choice of human capital investment and fertility. The resulting Stackelberg-equilibrium solution is thus inferred from:

$dW^i(t+1)/dw^i_{t+1} = [\partial W^i(t+1)/\partial H^i_{t+1}]\ [\partial H^i_{t+1}/\partial w^i_{t+1}] + \partial W^i(t+1)/\partial w^i_{t+1}$

$= d^i_1(t+1)^{-\sigma} c^i_1(t+1) A^i (S^i_t)^\gamma (\partial h^i_t/\partial w^i_{t+1}) - d^i_1(t+1)^{-\sigma} A^i h^i_t (S^i_t)^\gamma = 0$, where $d^i_1(t+1) \equiv (1 - v^i n^i_t - \theta^i h^i_t n^i_t - w^i_{t+1}\lambda^i_{t+1})$, $\lambda^i_{t+1} \equiv [H^i_{t+1} /(\overline{H}^i + H^i_{t+1})]$, and $c^i_1(t+1) \equiv (1 - v^i n^i_t - \theta^i h^i_t n^i_t - w^i_{t+1})$. In a growth equilibrium steady state, $d^i_1(t+1) = c^i_1(t+1)$. The optimal support rate, w^{i*}, equates the marginal cost and benefit to grown-up children from rewarding their parents for the earning capacity they helped create, subject to the "reaction function" $\{h^i_t, w^i_{t+1}\}$ governing the parents' investment decision ($\partial h^i_t/\partial w^i_{t+1}$).

Under our heterogeneity restriction, the optimal support rates w^{i*} become **identical** across family groups at both the stagnant and growth equilibrium. Consequently, the comparative dynamics simulations of a model with endogenous w become qualitatively identical to those reported in Table 1, where w is treated as a fixed, but identical across family groups.

Our simulations also show that the optimal value of w^* falls following any parametric shocks that produce takeoffs from stagnant- to growth-equilibrium steady states, essentially because the continuous growth in the level of offspring's human capital assets lowers the rate of return per unit of asset demanded in compensation by altruistic parents. The simulations also indicate that w^* falls with A^1, 1/v, and B in the growth steady state. Similar results are obtained in the stagnant steady state, except that a higher A^1 raises w* in that state. Shifts in γ and $\overline{H}^i$ have no effect on w^*.

D. Variables used and summary statistics

Variable	Description	Mean [Std. Dev.]
SD-FERT	Standard deviation of the distribution of surviving children per female ≥ 40	2.520 [0.306]
AV-FERT	Average of the distribution of surviving children per female ≥ 40	4.179 [1.161]
SD-SCHYR	Standard deviation of the distribution of schooling years in the population ≥ 15	3.684 [0.806]
AV-SCHYR	Average of the distribution of schooling years in the population ≥ 15	4.888 [2.755]
GINI*	Gini coefficient	37.76 [7.948]
QUINT*	Share of total income received by the top relative to the bottom quintile of families in the population	8.826 [5.259]
RGDP	Real per-capita income	6340 [5960]
GOV	GDP shares of government spending	19.53 [8.821]
GED	Share of government educational expenses in GDP	5.346 [1.550]

* We calculate GINI and QUINT exclusively based on household income data reported in Dollar and Kraay (2001), excluding observations based on personal income, personal expenditures, or household expenditure data.

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ENDNOTES

[1] Some studies favor the Kuznets hypothesis: e.g., Lindert and Williamson (1985) and Barro (2000). Others reject it, or find no systematic relation, e.g., Anand and Kanbur (1993), Fields (1990), Fields and Jakubson (1994), and Deininger and Squire (1998). Studies of the relation between the **rate of income growth** and income inequality also report mixed results: Persson and Tabellini (1994), Alesina and Rodrik (1994), and Deininger and Squire (1998) find a negative relation; Forbes (2000) finds a positive one; Barro (2000) finds that higher inequality lowers the growth rate in poor countries while raising it in rich countries, while Banerjee and Duflo (2000) find an inverted-U relation between the two.

[2] Models supporting Kuznets' causality direction rely on, e.g., structural shifts in a two-sector model (Kuznets 1955, 1963, Anand and Kanbur 1993); skill-biased technical progress (Eicher 1996, Aghion et al. 1999); and organizational changes (Kremer and Maskin 1996, Lindbeck and Snower 1997, Acemoglu 1999). Models favoring causality going from inequality to growth rely on, e.g., credit market imperfections (Loury 1981, Galor and Zeira 1993, Banerjee and Newman 1993, Benabou 1996, Durlauf 1996, and Galor and Moav 2004); political economy changes (Venieris and Gupta 1986, Alesina and Perotti 1996, Benhabib and Rustichini 1996); and fertility changes by income (Kremer and Chen 2002, and De la Croix and Doepke 2003).

[3] Lucas (1988) also considers spillover effects in goods production, stemming from the average human capital level in a representative-agent model. Tamura (1991) applies a similar spillover effect in human capital production, which results, however, in full income-convergence. Zhong (1998), and Ehrlich and Yuen (2000) develop a framework similar to ours, but abstract from the dynamic ramifications of the joint fertility and investment in human capital choices.

[4] Becker (1973) and Burdett & Coles (1997) offer evidence supporting positive assortative mating by intelligence, education, and related characteristics. Note that our assumed fixed distribution of family types still allows for family-group mobility over time, since groups with initially low earning capacity may move closer to, or even overtake, group 1 in the course of development (see fn 16). We can also allow for individual mobility as well by introducing stochastic deviations in, say, inherited ability (A) within subgroups (k) of family type (i), using a stochastic specification similar to that of Becker and Tomes (1979): $A^{k,i}_{t+1} = \phi A^{k,i}_{t} + (1-\phi)A^{i} + \varepsilon^{k,i}_{t+1}$, where $A^{k,i}_{t+1}$ is the ability level of generation (t+1) in subgroup k, A^{i} is the mean ability level of group i, ϕ (<1) is a positive weight, and $\varepsilon^{k,i}_{t+1}$ is a stochastic variable, iid, with zero mean and constant variance. Assuming positive assortative mating within subgroups, we obtain regression towards the mean ability level A^{i} within subgroups. Our model's basic implications hold under this extension as well.

[5] We also abstract from strategic group behavior designed to benefit from spillover effects.

[6] While there are other ways to model social interaction in the multiple-family-groups case, we can show that the "odds-ratio" (N^1/N^i) by which E^i is weighted, or any linear transformation thereof, is necessary to achieve a steady-state equilibrium solution in this deterministic setting. Alternative

"weights", such as $N^1_t/(N^1_t+N^i_t)$, $N^1/\Sigma_{j=1}^{I} N^j$, or $N^1/\Sigma_{j=2}^{I} N^j$, would fail to produce steady state solutions for our endogenous population shares, even in the two-family-group case. Chiu (1998) uses a similar weight in his social interaction function.

[7] Reliance on such inter-generational transfers going from children to parents plays an important role in our analysis because it guarantees the existence of interior solutions for both fertility (n^i) and investment in education (h^i) in the stagnant -, as well as the growth-equilibrium steady states for all agents, and thus for all our inequality measures (see footnote 14).

[8] In the stagnant steady state, two steady states may exist, but only one is locally stable (see fn. 11)

[9] For example, in the log-utility case (σ =1), it can be shown that the share of income devoted to both raising children and supporting old parents, v^i and w^i, must satisfy the equality $(1- w^1)/v^1 = (1- w^i)/v^i$ to assure equal fertility rates $n^1=n^i$ in a growth steady state (see fn 15). This condition is generally satisfied only if $w^1=w^i$ and $v^1=v^i$. Moreover, if we treat w^1 and w^i as **endogenous** variables, the fertility rates, n^1 and n^i, become functions of v^1 and v^i, respectively, given all other parameters. In Appendix C we show that under our heterogeneity restriction, if we set $v^1=v^i$, both optimal old-age support rates and fertility rates equalize. If we allow a pair of values for v^1 and v^i to be unequal (given all other parameters), however, this cannot generally support an equilibrium solution for w^1 and w^i that also satisfies $n^1=n^i$, regardless of the value of σ.

[10] As propositions 3 and 4 below indicate, in any stable equilibrium steady state, all families spend the same proportion of their potential income on the quantity (vn) and quality (θh) of children. Thus all wind up with the same fertility level, despite their different income levels, because in equilibrium, a lower income ($Y^i_t=\overline{H}^i+H^i_t$) would be offset by a proportionately lower shadow price of fertility ($[v^i+\theta^i h^i_t]Y^i_t$), and the demand for children's quantity n^i becomes a function of the ratio of the two.

[11] In the log utility case, the SE value of $h^i(s)$ has an explicit solution under our heterogeneity restriction: $h^i(s) = \{\Omega - [\Omega^2 - 4(A^i/\theta^i)\, S^i(s)^\gamma\, v^i]^{1/2}\}/[2(A^i/\theta^i)\, S^i(s)^\gamma]$; where $\Omega \equiv \beta - (A^i/\theta^i)\, S^i(s)^\gamma\, v^i$, and $S^i(s) \equiv [(A^1/\theta^1)/(A^i/\theta^i)]^{(1/\gamma)}$. This value of $h^i(s)$ is in fact one of two solution candidates that satisfy the optimality conditions, but the other solution leads to an **unstable** equilibrium. The SE value $n^i(s)$ is given implicitly by $[v^i+\theta^i h^i(s)]n^i(s)/[\delta(1+\beta)] = 1 - [v^i+\theta^i h^i(s)]n^i(s) - w^i A^i h^i(s) S^i(s)^\gamma$.

[12] Imposing the condition $A^i h^i (S^i)^\gamma = A^1 h^1$ (Proposition 1) and equation (10) on equations (6) and (7), the first-order conditions with respect to n and θh, and thus the equilibrium rates of return on children become **identical** across all family groups under our heterogeneity restriction. From proposition 1 and equation (11), $h^1/h^i = \theta^i/\theta^1$, we then have $S^i = (A^1h^1/A^ih^i)^{1/\gamma} = (e^1/e^i)^{(1/\gamma)}$. Note that the same result holds at the **growth equilibrium** steady state: the first part of equation (10) holds in the limit, or $E^i(g) = H^1_t/H^i_t$, as H^i_t tends to infinity.

[13] More specifically, $\partial G/\partial x = (1-1/E^i)(1-P^i \cdot S^i)\, \partial S^i/\partial x + S^i(P^i+1/E^i)^2\, \partial E/\partial x$ in any **equilibrium** position, where x is one of our parameters. In a stagnant state, a rise in (e^1/e^i) unambiguously raises

$S^i(s)$, while not affecting $E^i(s)$. Thus G(s) will rise or fall depending on whether $P^i(s)\cdot S^i(s)$ is smaller or bigger than 1. An increase in $\overline{H}^1/\overline{H}^i$ will not affect $S^i(s)$ but will raise unambiguously $E^i(s)$ and G(s). In a growth steady state, we cannot make symmetrical predictions because an increase in (e^1/e^i), for example, may also affect $E^i(g)$, as our analysis in section III.B indicates.

[14] A subtle point about proposition 3 is that it holds strictly under our benchmark case, which allows for intergenerational transfers (w>0). If we rely on altruism as the sole operating motive for parents (i.e., w=0), it can be shown that the only stable SE steady state requires a corner solution in human capital investments, or $h^i=0$, for all family groups (see Appendix B). There are then no spillover effects that link family groups 1 and i>1. The family-income inequality would then trivially become the endowment ratio in the SE steady state, $E^i(s)= \overline{H}^1/\overline{H}^i$, as is the endogenous outcome in our benchmark case, but our income-group inequality index and the Gini coefficient would be indeterminable in that state.

[15] In the log utility case, the growth steady state values $h^i(g)$ and $n^i(g)$ have explicit solutions:
$h^i(g) = 2v^i/[\theta^i(\beta-1)]$; and $n^i(g) = \delta(\beta-1)(1-w^i)/[v^i + v^i\delta(\beta+1)]$.

[16] There also is the possibility of mixed cases. For example, a technological shock reaches first family 1 (case b), but government subsidization of education targets family i (case c). In this case, the income inequality paths would have an inverted U shape, but the inequality levels would be lower in the growth equilibrium steady state relative to the SE state. Alternatively, if a reduction in θ^i affects family i many periods ahead of family 1, or by a sufficiently greater proportion, so that e^1/e^i actually falls, family i can **overtake** family 1, and become the "leading family" in terms of income-generating capacity. Income inequality will then reach a minimum at the point of overtaking, but will rise afterwards until it converges on its GE steady-state level. In this case the time path of income inequality will assume an S shape.

[17] De Gregorio and Lee (2002) support this expectation. They estimate a positive relationship between inequality in educational attainments and income inequality.

[18] The Barro-Lee study reports average schooling years for four schooling levels in the population age 15 and up (zero, primary, secondary, and higher) and their population shares. We calculate the mean and standard deviation of this distribution for each country in all sample years.

[19] Also, regression results obtained when using a polynomial of RGDP of the 4^{th}, 5^{th}, and 6^{th} order showed the same pattern as in all panels of Figure 3.

Table 1: Simulating Comparative Dynamic Effects of Parameter Changes in a Two-agent Economy

Part 1. Stagnant Equilibrium

A^1/θ^1	A^2/θ^2	$\overline{H}^1$	$B^1(B^2)$	$v^1(v^2)$	$w^1(w^2)$	γ	$n^1(n^2)$	$Y^1=\overline{H}^1+H^1$	$Y^2=\overline{H}^2+H^2$	E	S	$P=N^1/N^2$	Gini
2/1	1/1.01	50	.1	.05	.01	.4	6.958	54.905	1.0981	50	5.7993	.1160	.7490
3/1	1/1.01	50	.1	.05	.01	.4	1.879	453.67	9.0734	50	15.981	.3196	.6989
3/1	**1.5/1.01**	50	.1	.05	.01	.4	1.879	453.67	9.0734	50	5.7993	.1160	.7490
2/1	**1.5/1.01**	50	.1	.05	.01	.4	6.958	54.905	1.0981	50	2.1045	.0421	.6375
2/1	1/1.01	**60**	.1	.05	.01	.4	6.958	65.886	1.0981	60	5.7993	.0967	.7648
2/1	1/1.01	50	**.15**	.05	.01	.4	6.999	54.856	1.0971	50	5.7993	.1160	.7490
2/1	1/1.01	50	.1	**.055**	.01	.4	6.249	55.597	1.1119	50	5.7993	.1160	.7490
2/1	1/1.01	50	.1	.05	**.015**	.4	6.933	54.955	1.0991	50	5.7993	.1160	.7490
2/1	1/1.01	50	.1	.05	.01	**.45**	6.958	54.905	1.0981	50	4.7704	.0954	.7396

Part 2. Growth Equilibrium

A^1/θ^1	A^2/θ^2	$B^1(B^2)$	$v^1(v^2)$	$w^1(w^2)$	γ	$n^1(n^2)$	h^1	h^2	$a^1=A^1h^1$	E	S	$P=N^1/N^2$	Gini
30/1	15/1.01	.1	.05	.01	.4	1.227	.4886	.4837	14.657	50	5.7993	.1160	.7490
40/1	15/1.01	.1	.05	.01	.4	1.229	.4885	.4836	19.542	113.9	11.905	.1045	.8279
40/1	**20/1.01**	.1	.05	.01	.4	1.229	.4885	.4836	19.542	50	5.7993	.1160	.7490
30/1	**20/1.01**	.1	.05	.01	.4	1.227	.4886	.4837	14.657	21.9	2.8251	.1290	.6243
30/1	15/1.01	**.15**	.05	.01	.4	1.233	.4866	.4818	14.598	50	5.7993	.1160	.7490
30/1	15/1.01	.1	**.055**	.01	.4	1.115	.5375	.5321	16.125	50	5.7993	.1160	.7490
30/1	15/1.01	.1	.05	**.015**	.4	1.218	.4905	.4856	14.715	50	5.7993	.1160	.7490
30/1	15/1.01	.1	.05	.01	**.45**	1.227	.4886	.4837	14.657	41.2	4.7704	.1158	.7229

Part 3. Takeoff Triggers

	A^1	A^2	θ^1	θ^2	$v^1(v^2)$	$w^1(w^2)$	$n^1(n^2)$	h^1	h^2	E	S	$P=N^1/N^2$	Gini
(1) (SE)	2	1	1	1.01	.05	.01	6.958	.0447	.0442	50	5.7993	.1160	.7490
(GE)	**30**	**15**	1	1.01	.05	.01	1.227	.4886	.4837	50	5.7993	.1160	.7490
(2) (SE)	2	1	1	1.01	.05	.01	6.958	.0447	.0442	50	5.7993	.1160	.7490
(GE)	**30**	1	1	1.01	.05	.01	1.227	.4886	.4837	4.5E+7	5053.6	1.1E–4	.9997
(3) (SE)	3	1	1	1.01	.05	.01	1.879	.2966	.2937	50	15.981	.3196	.6989
(GE)	3	1	**0.5**	**0.505**	.05	.01	1.214	.9771	.9674	50	15.981	.3196	.6989
(4) (SE)	3	1	1	1.01	.05	.01	1.879	.2966	.2937	50	15.981	.3196	.6989
(GE)	3	1	**0.5**	**0.5**	.05	.01	1.214	.9771	.9771	48.5	15.588	.3211	.6967
(5) (SE)	4.8	1	1	1.01	.05	.01	3.979	.1150	.1138	50	51.750	1.035	.4724
(GE)	4.8	1	1	1.01	**.06**	.01	1.009	.5865	.5807	50	51.750	1.035	.4724
(6) (SE)	4.8	1	1	1.01	.05	.01	3.979	.1150	.1138	50	51.750	1.035	.4724
(GE)	4.8	1	1	1.01	.05	**.05**	1.152	.4964	.4915	50	51.750	1.035	.4724

Note: Parameters values that deviate from our benchmark values are presented in bold print. In this table we treat w as exogenous. The comparative dynamic results are found to be qualitatively identical to those reported in parts 1 and 2 when we treat w as endogenous, using the analysis in Appendix C.

Part 1. Comparative dynamics in the stagnant steady state are simulated by changing A^1/θ^1, A^2/θ^2, $\overline{H}^1$, $B^1(=B^2)$, $v^1(=v^2)$, $w^1(=w^2)$, or γ, holding constant the values of all other parameters: $\overline{H}^2 = 1$, $\sigma = 0.98$, $\delta = 0.9$, $\beta = 1.2$, and $\alpha = 1$.

Part 2. Comparative dynamics in the growth steady state are simulated by changing A^1/θ^1, A^2/θ^2, $B^1(=B^2)$, $v^1(=v^2)$, $w^1(=w^2)$, or γ, holding constant $\sigma = 0.98$, $\delta = 0.9$, $\beta = 1.2$, and $\alpha = 1$.

Part 3. Simulations show the impact of uniform (proportionate) and non-uniform changes in A^i and θ^i, as well as in the common levels of $v^1=v^i$ and $w^1=w^2$ at both the SE and GE steady states, holding constant: $\overline{H}^1 = 50$, $\overline{H}^2 = 1$, $\sigma = 0.98$, $\delta = 0.9$, $\gamma = 0.4$, $\beta = 1.2$, $\alpha = 1$, and $B^1 = B^2 = 0.1$.

Table 2 Fertility Inequality Regressions

Dependent Variable: SD_FERT

	Model 1	Model 2	Model 3	Model 4	Model 5
	OLS	Country Random Effects	Country Random Effects	Country Random Effects	Country RE & Year Dummies
Intercept	1.776852	2.087804	2.165747	2.181626	1.162350
	7.45	10.54	9.72	10.53	2.94
RGDP	0.000410	0.000249	0.000235	0.000270	0.000376
	2.23	1.59	1.49	1.83	2.27
$RGDP^2$	-8.49E-08	-5.75E-08	-5.62E-08	-5.59E-08	-8.13E-08
	-1.90	-1.60	-1.56	-1.66	-2.13
$RGDP^3$	5.08E-12	3.40E-12	3.40E-12	3.25E-12	5.07E-12
	1.64	1.44	1.44	1.47	2.00
AV_FERT	0.056786	0.042325	0.046133	0.134551	0.180190
	1.88	2.14	2.27	4.18	3.84
GOV			-0.003653	-0.004215	-0.005115
			-0.69	-0.87	-1.00
T				-0.013374	
				-3.46	
Adj. R^2	0.0854	0.1637	0.1959	0.2824	0.4373
N	72	72	72	72	72

Notes: The dependent variable is the standard deviation of the distribution of surviving children per woman age 40 and over. Data sources are the World Fertility Surveys and the Demographic and Health Surveys (various years). Rows show the estimated coefficients (β) and their t-statistics (β/S_β). This table's regressions employ a random effects specification to account for missing idiosyncratic variables, because the number of observations per country is small. No serial correlation correction is needed, since the Cochran-Orcutt test rejects the existence of an AR(1) serial correlation in Model 3.

Table 3. Education Attainment Inequality Regressions

Dependent Variable: SD_SCHYR

	Model 1	Model 2	Model 3	Model 4	Model 5
	OLS	Country Fixed Effects	Country Fixed Effects	Country Fixed Effects	Country& Year Fixed Effects
Intercept	2.634408	2.074857#	1.957999#	2.512515#	2.837973#
	37.43				
RGDP	9.29E-05	-3.79E-05	-3.13E-05	-4.36E-05	-5.95E-05
	2.71	-1.02	-0.84	-1.32	-1.75
$RGDP^2$	-1.20E-08	4.29E-09	4.01E-09	2.75E-09	3.34E-09
	-3.79	1.48	1.39	1.07	1.29
$RGDP^3$	3.03E-13	-1.59E-13	-1.54E-13	-1.19E-13	-1.24E-13
	3.48	-2.21	-2.15	-1.87	-1.94
AV_SCHYR	0.205215	0.351542	0.342305	0.111584	0.115726
	11.97	18.40	17.66	4.50	4.59
GOV			0.006915	0.000617	-0.000855
			2.51	0.25	-0.33
T				0.029080	
				12.92	
Adj. R^2	0.3320	0.4891	0.4943	0.6022	0.6070
N	721	721	721	721	721

Notes: The dependent variable is the standard deviation in the distribution of schooling years attained in the population age 15 and over. The data source is Barro and Lee (2000). Rows show the estimated coefficients (β) and their t-statistics (β/S_β). # The Intercept coefficients represent mean values of all intercept terms. No serial correlation correction is needed, since data on the dependent variable are available every five years and the Cochran-Orcutt test rejects the existence of an AR(1) serial correlation in Model 3.

Table 4. Income Inequality Regressions: GINI

Dependent Variable: GINI

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
	OLS	Country Fixed Effects	Country Fixed Effects	Country Fixed Effects	Country& Year Fixed Effects	OLS
Intercept	41.948920 26.34	35.729530#	40.193390#	39.392580#	58.906320#	42.198848 28.69
RGDP	0.001583 2.94	0.001095 2.62	0.000873 2.02	0.001081 2.03	0.001077 1.85	0.001645 3.28
$RGDP^2$	-2.45E-07 -5.22	-1.08E-07 -3.54	-9.47E-08 -3.00	-1.04E-07 -3.00	-1.01E-07 -2.68	-2.54E-07 -5.79
$RGDP^3$	7.04E-12 5.96	2.81E-12 3.99	2.51E-12 3.43	2.70E-12 3.43	2.61E-12 3.05	7.27E-12 6.61
GOV			-0.181998 -2.19	-0.170141 -2.00	-0.216757 -2.22	
T				-0.027090 -0.66		
Adj. R^2	0.4108	0.0691	0.0916	0.0932	0.2248	0.4691
N	318	318	310	310	310	305

Notes: The dependent variable is the GINI coefficient, based on household income data. The data source is Dollar and Kraay (2001). Rows show the estimated coefficients (β) and their t-statistics (β/S_β). # Coefficient represents the mean value of the intercept term. No serial correlation correction is needed, since the Cochran-Orcutt test rejects AR(1) serial correlation in Model 3.

Table 5. Income Inequality Regressions: QUINT

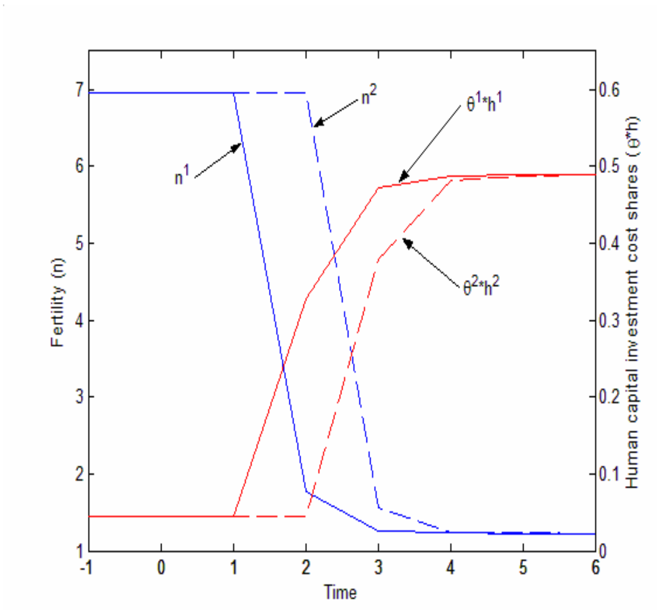
Dependent Variable: QUINT

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
	OLS	Country Fixed Effects	Country Fixed Effects	Country Fixed Effects	Country & Year Fixed Effects	OLS
Intercept	9.110258	4.513832#	2.445769#	2.290842#	25.019580#	9.167456
	7.52					7.67
RGDP	0.001440	0.001294	0.001437	0.001478	0.001313	0.001498
	3.47	3.82	4.10	3.37	2.76	3.61
$RGDP^2$	-1.80E-07	-1.00E-07	-1.13E-07	-1.15E-07	-1.07E-07	-1.87E-07
	-4.92	-3.98	-4.30	-3.95	-3.43	-5.11
$RGDP^3$	4.97E-12	2.32E-12	2.65E-12	2.69E-12	2.55E-12	5.13E-12
	5.34	3.96	4.30	4.02	3.55	5.54
GOV			0.101228	0.103520	0.077167	
			1.51	1.50	0.99	
T				-0.005237		
				-0.16		
Adj. R^2	0.2498	0.0663	0.0787	0.0788	0.2278	0.2695
N	289	289	281	281	281	276

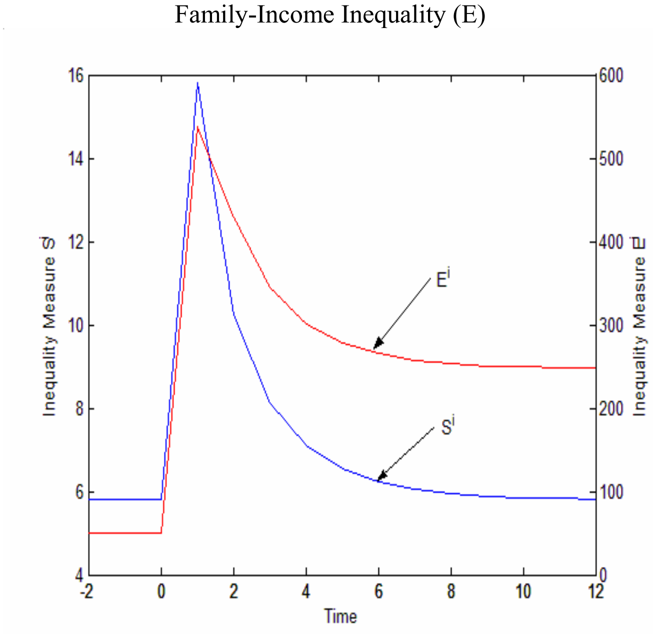
Notes: The dependent variable is the share of total income received by the top, relative to the bottom, quintile of families in the population. The data source is Dollar and Kraay (2001), and only household income data are used. Rows show the estimated coefficients (β) and their t-statistics (β/S_β). # Coefficient represents the mean value of the intercept terms. No serial correlation correction is needed, since the Cochran-Orcutt test rejects AR(1) serial correlation in Model 3.

Figure 1. Simulated time paths when a uniform shock affects family 1 ahead of family 2 in the two-family case

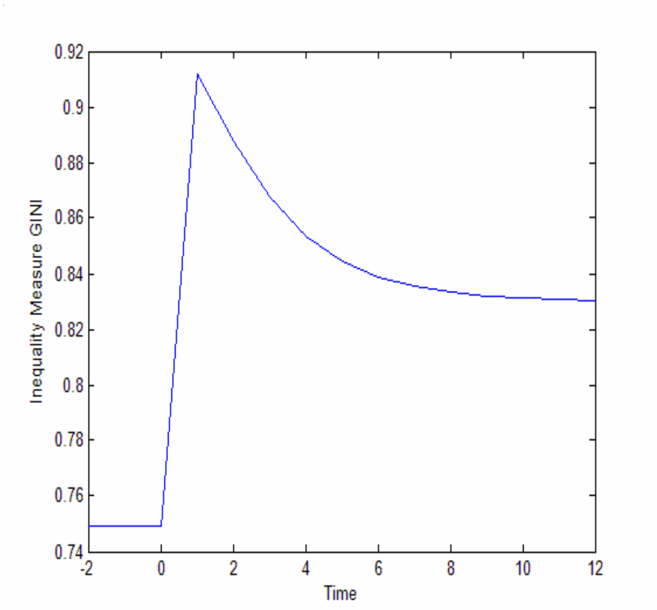
a. Fertility (n) and Human Capital Investment Cost Share (θh)



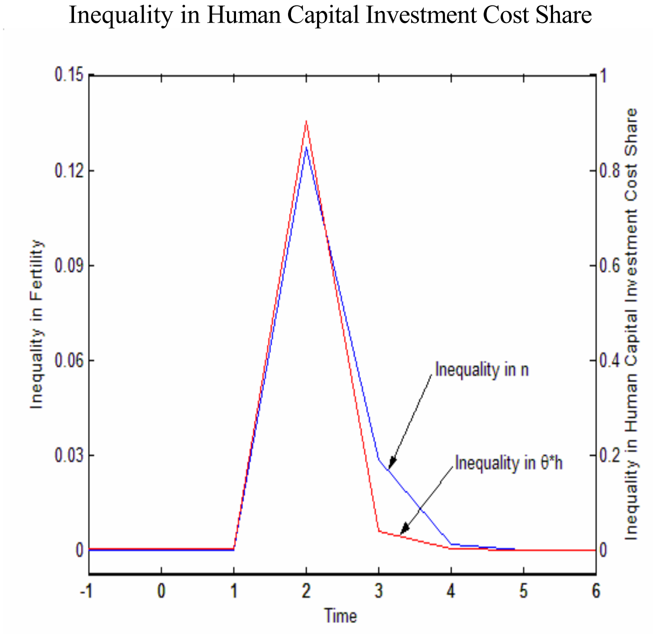
b. Income-Group Inequality (S) and Family-Income Inequality (E)



c. Gini Coefficient (G)



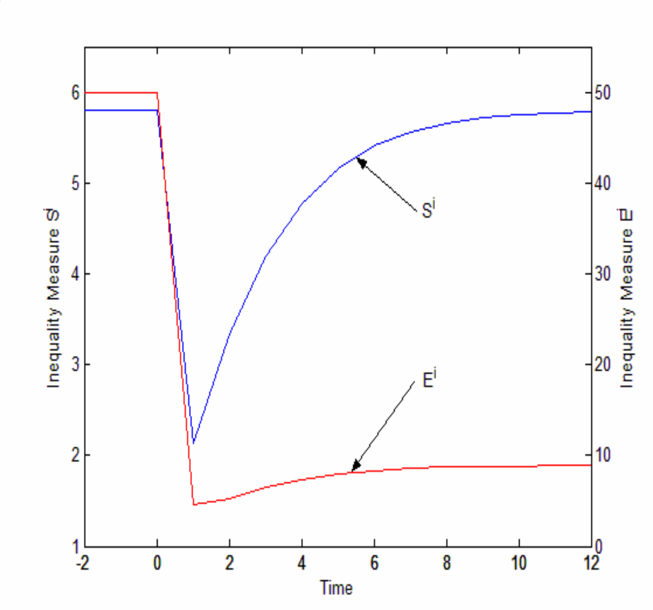
d. Inequality in Fertility and Inequality in Human Capital Investment Cost Share



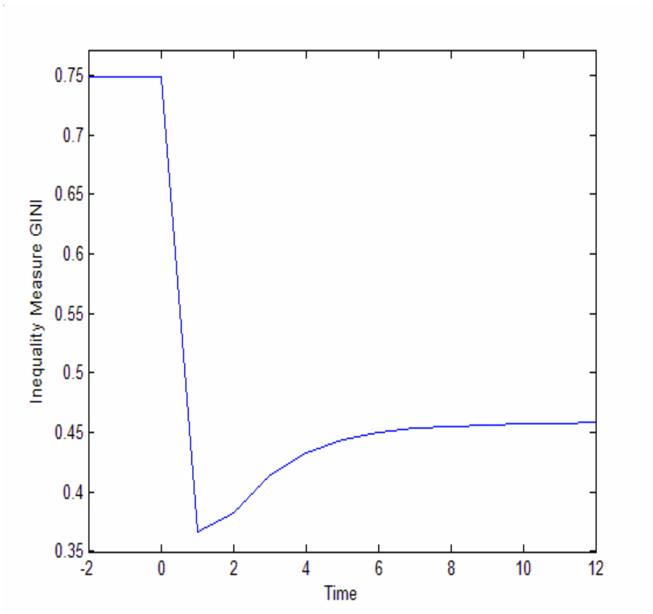
Note: Parameter values used in these simulations are: $\theta^1=1$, $\theta^2=1.01$, $H^1_0=50$, $H^2_0 = 1$, $B^1=B^2=0.1$, $w^1=w^2=0.01$, $v^1=v^2=0.05$, $\gamma = 0.4$, $\sigma = 0.98$, $\delta = 0.9$, $\beta = 1.2$, $\alpha = 1$. Prior to period 1, the economy is in a stable SE steady state, with $A^1=2$ and $A^2=1$. In period 1, family 1 alone experiences a once-and-for-all permanent increase in A^1 to 30. In period 2, family 2 also experiences an equi-proportional increase in A^2 to 15.

Figure 2. Simulated time paths when a uniform shock affects family 2 ahead of family 1 in the two-family case

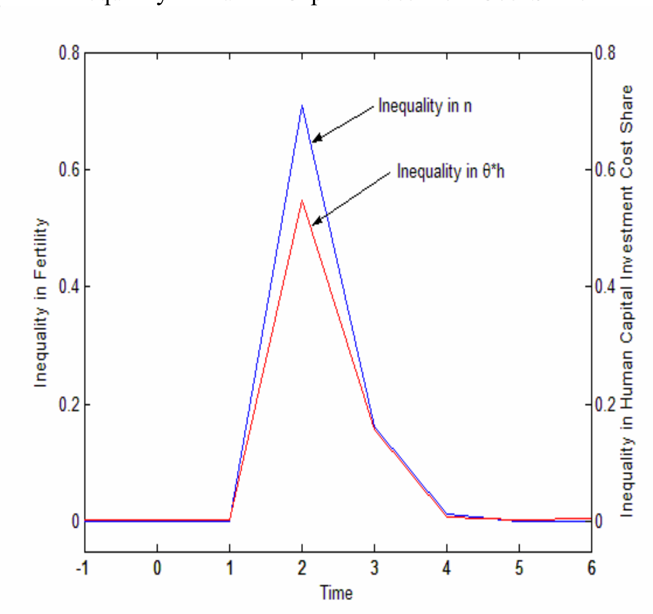
a. Income-Group Inequality (S) and
Family-Income Inequality (E)



b. Gini Coefficient (G)

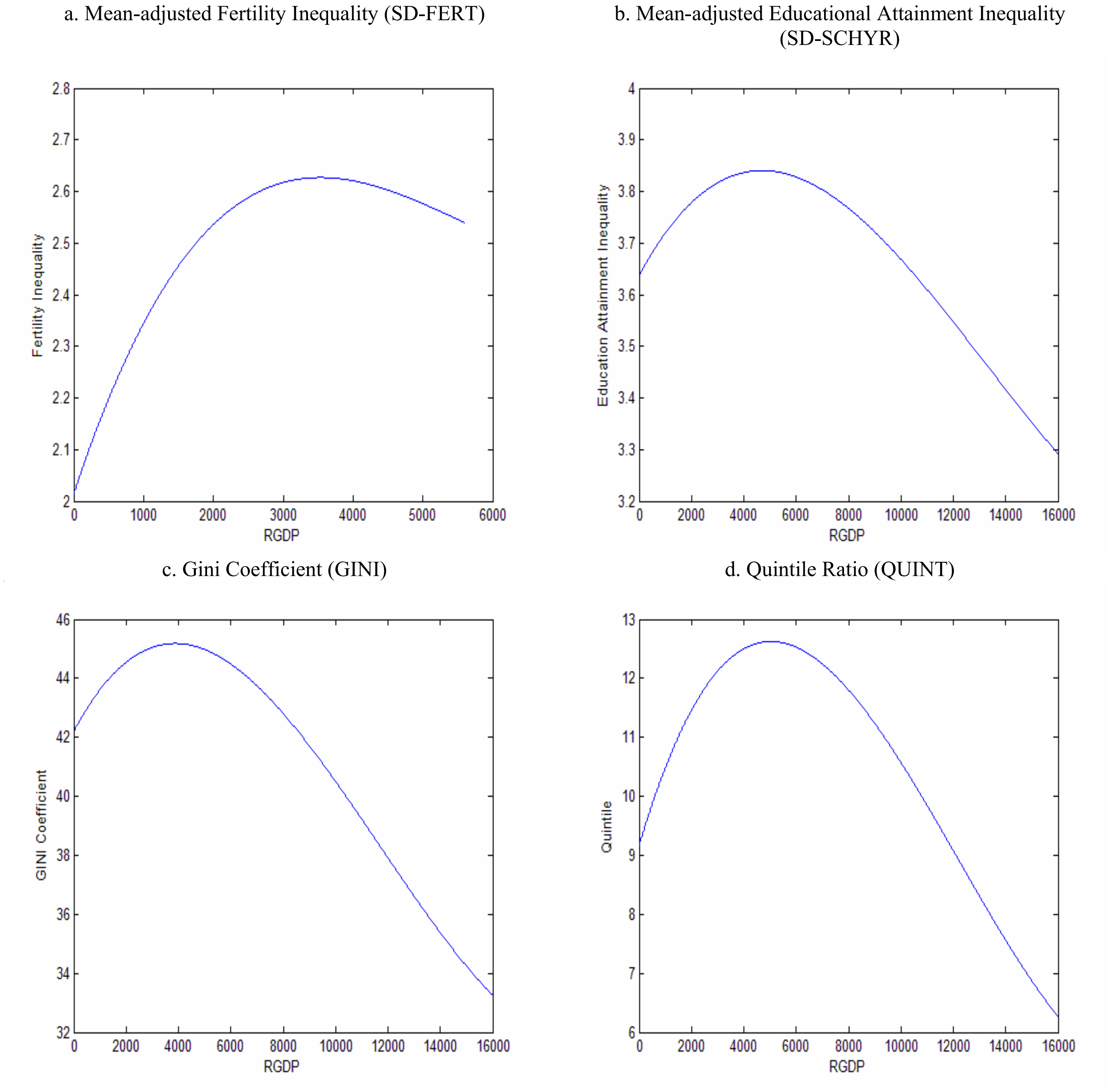


c. Inequality in Fertility and
Inequality in Human Capital Investment Cost Share



Note: Parameter values used in the simulations for Figure 2: $A^1=2$, $A^2=1$, $H^1_0=50$, $H^2_0 = 1$, $B^1=B^2=0.1$, $w^1=w^2=0.01$, $v^1=v^2=0.05$, $\gamma = 0.4$, $\sigma = 0.98$, $\delta = 0.9$, $\beta = 1.2$, $\alpha = 1$. Prior to period 1, the economy is in a stable SE steady state with $\theta^1=1$ and $\theta^2=1.01$. In period 1, family 2 alone experiences a once-and-for-all reduction in θ^2 to 1.01/15. In period 2, family 1 then experiences an equi-proportional reduction in θ^1 to 1/15.

Figure 3. Fitted Lines from the Regression Results



Note: Panels a, b, c, and d are based on the regression results of Model 1 in Tables 2 and 3, and Model 6 in Tables 4 and 5, respectively. The RGDP values on the x-axes of all panels cover 90% of the observations on RGDP used in our regressions.