

# The Production of Cognitive Achievement in Children: Home, School and Racial Test Score Gaps

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## Abstract

This paper studies the determinants of children's scores on tests of cognitive achievement in math and reading. Using rich longitudinal data on test scores, home environments, and schools, we implement alternative specifications for the cognitive achievement production function that allow achievement to depend on the entire history of lagged home and school inputs as well as on parents' ability and unobserved endowments. The empirical results show that both contemporaneous and lagged inputs matter in the production of current achievement and that it is important to allow for unobserved endowment effects. We use cross-validation methods to select among competing specifications and find support for a variant of a value-added model of the production function augmented to include information on lagged inputs. Using this specification, we study the sources of test score gaps between black, white and Hispanic children. The estimated model captures key patterns in the data, such as the widening of minority-white test score gaps with age and differences in the gap pattern between Hispanics and blacks. We find that differences in mother's ability (as measured by AFQT) accounts for roughly half of the test score gap. However, home inputs also account for a significant proportion. Equalizing home inputs at the average levels of white children would close the black-white test score gaps in math and reading by about 25% and the Hispanic-white gap by about 30%.

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# 1 Introduction

It is well documented that scores on cognitive tests taken by adolescents are predictive of future labor market outcomes, such as educational attainment and earnings.<sup>1</sup> Even test scores at age seven have been shown to be correlated with measures of labor market success.<sup>2</sup> These findings have led many researchers to assign a large role to "premarket factors" in explaining adult earnings inequality, where premarket factors are broadly interpreted to represent endowed ability, the effects of family background and the influence of schools.

Premarket factors are also thought to be an important part of the explanation for racial disparities in test score performance and labor market outcomes.<sup>3</sup> Although it is conceivable that test score gaps could arise from differential investment in children based on expectations about future labor market returns (a post market rather than a premarket factor), Carniero, Heckman and Masterov (2002) argue that this is unlikely. They document that test score gaps between white and black children already emerge by the age of school entry and tend to widen with age.<sup>4</sup> Although there has been some narrowing in the overall black-white and Hispanic-white test score gaps since the 1970's, there is still a substantial disparity with black children scoring about 15-25% lower than whites on average and Hispanic children about 10% lower.<sup>5</sup>

The belief that eliminating racial differences in test score performance would reduce inequality in labor market outcomes is a major motivation for the extensive, multidisciplinary

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<sup>1</sup>See e.g. Leibowitz (1974), Murnane, Willett and Levy (1995), Neal and Johnson (1996), Keane and Wolpin (1997), and Cameron and Heckman (1998).

<sup>2</sup>Robertson and Symons (1996) find that age seven test scores predict occupational choices, and Currie and Thomas (1999) document their correlation with adult educational and labor market outcomes. These studies are based on data from the British National Child Development Survey.

<sup>3</sup>Neal and Johnson (1996).

<sup>4</sup>See also related discussion in Levitt and Fryer (2002), Phillips, Crouse and Ralph (1998), and section 3 of this paper. There is, however, some debate over whether test scores widen as children progress through school grades (Ludwig (2003)). Carniero and Heckman (2003) discuss gap patterns by age in noncognitive test score measures.

<sup>5</sup>See Jencks and Phillips (1998), Cook and Evans (2000) for a discussion of trends in scores on NAEP (National Assessment of Educational Progress) tests. Hedges and Nowell (1998, 1999) analyze data from six surveys that include EEO (Equal Educational Opportunity Data), National Longitudinal Study of the High School Class of 1972, High School and Beyond, National Longitudinal Survey of Youth (1979), National Education Longitudinal Study of 1988, and NAEP.

literature aimed at understanding the determinants of children's test scores.<sup>6</sup> A large body of research examines the role of parental characteristics, the early home environment, and school quality in producing cognitive skills. However, these studies have not yet led to a consensus view on which inputs increase children's achievement and to what extent, or on the relative contribution of home inputs, school inputs and endowments in accounting for racial/ethnic differences in achievement. A leading candidate for explaining why studies tend to reach very different conclusions, even when based on the same datasets, is the wide variety of empirical specifications adopted in the empirical literature (Krueger, 2003, Todd and Wolpin, 2003).

Ideally, in analyzing cognitive achievement of children, it would be useful to have data on all past and present home and school inputs as well as information on children's heritable endowments. However, no dataset is that comprehensive, and researchers have had to confront problems of missing or imprecisely measured variables. One approach explicitly recognizes the presence of omitted variables and develops estimators that allow for them. For example, Murnane, Maynard and Ohls (1981) use school fixed effects to address the problem of missing school inputs, under the assumption that children within the same school receive the same inputs. Rosenzweig and Wolpin (1994) and Altonji and Dunn (1996) address the same problem using sibling fixed effects. An alternative approach that is commonly taken when the data lack information on historical input measures, is to adopt a value added specification that assumes that a previous test score is a sufficient statistic for the missing historical inputs. Yet another remedy to the problem of missing data on inputs is to implicitly substitute input demand functions in place of the missing inputs.<sup>7</sup>

This paper has two main goals: to quantify the impact of home inputs, school inputs and mother's ability on children's achievement and to analyze the relative contribution of each factor in accounting for racial/ethnic test score gaps. The main innovation relative to the earlier literature is to implement a cumulative production function for children's cognitive achievement that allows achievement at a given age to depend on the lifetime history of family and school inputs as well as on mother's ability and heritable endowments. Our modeling approach builds on Boardman and Murnane (1979), who were the first to formalize a cumulative model of the cognitive achievement production function and to discuss its

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<sup>6</sup>A review of the literature can be found in Todd and Wolpin (2003).

<sup>7</sup>See section four below for a discussion of this approach.

potential implementation in cross-section and panel data settings.<sup>8</sup> It also builds on Todd and Wolpin (2003), which surveys the literature on estimating production functions for cognitive achievement and discusses the identification assumptions of alternative estimators. Our work is complimentary to recent work by Cunha and Heckman (2003) that extends the production function framework to incorporate the development of noncognitive skills and their influence on cognitive skill development. They adopt a value-added specification for the joint formation of cognitive and noncognitive skills, allowing for measurement error in skills and in home inputs. One interesting finding from their work is that noncognitive skills promote the formation of cognitive skills but not vice versa. They also find evidence for critical skill investment periods during which time the investment needs to be made to be effective, and they demonstrate the importance of early investments. As discussed later, some of the specifications for the cognitive achievement production function that we implement are consistent with their approach, although we do not explicitly model how noncognitive skills are formed along with cognitive skills.

An important issue in the estimation of the cognitive achievement production function studies is how to select among competing model specifications. In this paper, we resolve the model selection problem by applying cross-validation criteria in addition to conventional specification tests. Cross-validation methods find the model that performs best according to an out-of-sample root-mean-squared error (RMSE) criterion. The method is a useful alternative to conventional specification testing in situations where the models being compared are non-nested and/or when it is not clear which is the preferred null hypothesis model. Cross validation also seems particularly well suited to our intended use of estimated model to decompose test score gaps into components due to home and school environments. Specifically, when we use the estimated model to evaluate how much of the minority-white test score gaps would be closed if black or Hispanic children had the same home and school inputs as white children, we essentially perform an out-of-sample forecast. The cross-validation criterion evaluates the reliability of the model in out-of-sample forecasting.<sup>9</sup> The cross-validation results indicate support for an augmented value-added formulation of the production function.

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<sup>8</sup>We recently became aware of this insightful but overlooked paper.

<sup>9</sup>In a later section, we also discuss potential problems associated with using an atheoretical approach such as cross-validation as a tool for model selection.

Our analysis samples are drawn from the National Longitudinal Surveys of Labor Market Experience - Children Sample (NLSY79-CS) merged together with school quality data obtained from two sources: the Common Core Data (CCD) and the American Federation of Teachers (AFT). The NLSY79-CS data contain detailed longitudinal information on children's home environments and on child achievement as measured by scores on tests that are administered biannually. The CCD and AFT data are used to derive a time series of school quality, as measured by pupil-teacher ratios and teacher salary.

Using these data, we implement alternative specifications of the production function. The estimates strongly support the notion that skill accumulation is a cumulative process; both contemporaneous and past home inputs are highly significant determinants of test score outcomes. The effects of the school input variables on test scores are also statistically significant at conventional levels for most of the specifications considered, but they are imprecisely measured in models that incorporate fixed effects.

We use our estimates of the cognitive achievement production function parameters to examine the extent to which home input differences can account for racial disparities in test scores among African American, white and Hispanic children. Our work differs from earlier studies, in part, because our specifications allows for unobserved endowment effects and for the cumulative effects of lagged inputs.<sup>10</sup> The empirical results show that equalizing home input levels at the average level observed for white children would close about 25% of the black-white test score gap (in both math and reading) and 30% of the Hispanic-white test score gap.<sup>11</sup> We also find that the estimated cognitive achievement production function fits

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<sup>10</sup>For example, Cook and Evans (2000) decompose test score differences into components due to changing relative levels of parental education, changing levels of school quality, and a narrowing of within school gaps, using data from NAEP (National Assessment of Educational Progress). The NAEP dataset has the advantage of being a large representative sample with multiple observations per school, but it contains little information on childrens' home environments and is not longitudinal. Also, see Fuchs and Reklis (1994) for an analysis of the sources of racial math test score differences using state-level NAEP data.

<sup>11</sup>Our finding that home input gaps are important in accounting for racial test score gaps contrasts with findings reported in recent work by Levitt and Fryer (2004). That paper argues that home input gaps cannot account for black-white test score gaps, because home input gaps remain roughly constant over time whereas test score gaps widen with age. However, their specification assumes test scores depend only on current home inputs. A specification that allows for lagged inputs to have an effect can explain a widening black-white test score gap, because a constant home input gap over time implies a widening cumulative gap.

well the pattern of rising black-white test score gaps with age as well as differences in test score gap patterns between girls and boys.

This paper proceeds as follows. Section two of the paper proposes a conceptual framework for modeling the cognitive achievement production function and considers its empirical implementation. Section three describes our data sources and the variables used to represent home and school inputs into the production process. Section four presents estimates of the cognitive achievement production function obtained under alternative specifications and also present the cross-validation results. Section five uses the estimated cognitive achievement production function to evaluate the sources of racial disparities in test scores and section six concludes.

## 2 Alternative Approaches to Modeling and Estimating the Production Function for Achievement

In this section, we lay out a framework for modeling the cognitive achievement production function. It assumes that knowledge acquisition is a cumulative process by which current and past inputs are combined with a child’s genetic endowment of mental capacity (determined at conception) to produce a cognitive outcome.<sup>12</sup> Let  $A_{ija}$  denote the achievement for child  $i$  residing in household  $j$  at age  $a$  and  $Z_{ija}(a)$  the vector of all inputs applied at any time up until age  $a$ .<sup>13</sup> The child’s endowed mental capacity (ability) is represented by  $\mu_{ij0}$ . The achievement production function that relates test scores at age  $a$  to all prior investments in the child is given by

$$A_{ija} = A_a(Z_{ij}(a), \mu_{ij0}). \tag{1}$$

The empirical implementation of (1) is problematic because heritable endowments are not observed, data sets on inputs are incomplete (i.e. have incomplete input histories and/or missing inputs), inputs may be chosen endogenously with respect to unobserved endowments and scores on standardized tests measure achievement with error.

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<sup>12</sup>The production function framework was first formally modeled by Ben Porath (1967) in the context of an individual decision-maker choosing the level of (time and money) resources to devote to human capital investments. It has since served as the basis for much of the literature on skill acquisition in economics. Leibowitz (1974) was the first to extend this conception to home investments in children.

<sup>13</sup>We include in  $Z$  exogenous environmental factors, but for ease of notation we do not distinguish them.

Let  $T_{ija}$  be the test score measure that is observed and  $\varepsilon_{ija}$  a measurement error. Also, let  $X_{ija}$  and  $v_{ija}$  represent observed and unobserved inputs at age  $a$ . To arrive at an empirically implementable specification, assume that the production function is approximately linear in the inputs and in the unobserved endowment and that input effects do not depend on the child's age, but may depend on the age at which they were applied relative to the current age:

$$T_{ija} = X_{ija}\alpha_1 + X_{ija-1}\alpha_2 + \dots + X_{ij1}\alpha_a + \quad (2)$$

$$\beta_a\mu_{ij0} + v_{ija}\rho_1 + v_{ija-1}\rho_2 + \dots + v_{ij1}\rho_a + \varepsilon_{ija}. \quad (3)$$

We take this specification to be the most general structure. Data limitations have required researchers to place restrictions on (2). The following specifications of the production function and associated restrictions have been adopted or proposed in previous studies.

(i) *contemporaneous specification*

This specification relates an achievement test score to data only on contemporaneous inputs<sup>14</sup>:

$$T_{ija} = X_{ija}\alpha_1 + e_{ija} \quad (4)$$

where  $e_{ija}$  is a residual term that includes the effect of any omitted inputs, lagged inputs (observed and unobserved), endowments, and measurement error. The assumption required to consistently estimate  $\alpha_1$  is that all the omitted factors are orthogonal to the included input measure ( $E(e_{ija}|X_{ija}) = 0$ ).

(ii) *cumulative specification with orthogonal endowments and omitted inputs*

This specification expands the contemporaneous specification to include observable lagged inputs, but it maintains the assumption that any omitted inputs and endowments are orthogonal to the included inputs:

$$T_{ija} = X_{ija}\alpha_1 + X_{ija-1}\alpha_2 + \dots + X_{ij1}\alpha_a + e_{ija},$$

(iii) *Fixed effect specifications*

Fixed effect specifications provide ways of implementing either the contemporaneous model or the cumulative model in a way that allows for input choices to be endogenous with

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<sup>14</sup>Fryer and Levitt (2004) estimate a version of the contemporaneous model that assumes that inputs do not cumulate and does not allow for endogeneity of inputs.



respect to unobserved endowments. Two “fixed effect” estimators that are prominent in the literature use variation that occurs within families across siblings or within children across different ages.

*Within-child fixed effect estimators* are feasible when there are multiple observations on achievement outcomes and on inputs for a given child at different ages. Consider differencing the achievement test scores at two ages,  $a$  and  $a - 1$ ,

$$\begin{aligned} T_{ija} - T_{ija-1} &= (X_{ija} - X_{ija-1})\alpha_1 + (X_{ija-1} - X_{ija-2})\alpha_2 + \dots + \\ &\quad (X_{ij2} - X_{ij1})\alpha_{a-1} + X_{ij1}\alpha_a \\ &\quad + [\beta_a - \beta_{a'}]\mu_{ij0} + e_{ija} - e_{ija-1}. \end{aligned} \quad (5)$$

The parameters of (5) can be consistently estimated under the following assumptions. The first is that the impact of the endowment on achievement must be independent of age ( $\beta_a = \beta_{a'}$ ), so that differencing eliminates the endowment from (5). In that case, orthogonality between input choices and endowments need not be assumed. However, because any prior achievement outcome was known when later input decisions are made, it is necessary to assume that later input choices are invariant to prior own achievement outcomes. It is also necessary to assume that differenced included inputs are orthogonal to omitted inputs or that differenced omitted inputs are age-invariant (and are therefore eliminated by the differencing).

The *within-family fixed effect estimator* assumes that children of the same parents have a common heritable component. Without loss of generality, decompose the endowment into a family and a child-specific component, denoted by  $\mu_0^f$  and  $\mu_0^c$ . Rewriting (2) yields

$$T_{ija} = X_{ija}\alpha_1 + X_{ija-1}\alpha_2 + \dots + X_{ij1}\alpha_a + \beta_a\mu_{ij0}^f + \beta_a\mu_{ij0}^c + e_{ija},$$

where the effect of unobservable current and lagged inputs is subsumed into the residual  $e_{ija}$ . Consider the estimator in the case of two siblings ( $i$  and  $i'$ ) observed at the same age. Differencing the above equation yields

$$T_{ija} - T_{i'ja} = (X_{ija} - X_{i'ja})\alpha_1 + \dots + (X_{ij1} - X_{i'j1})\alpha_a + [\beta_a(\mu_{ij0}^c - \mu_{i'j0}^c) + e_{ija} - e_{i'ja}] \quad (6)$$

Consistent estimation of input effects by ols requires that inputs associated with any child not respond either to the own or the sibling child-specific endowment component. However, input choices may respond to the family endowment component. That is, parents who

perceive their children to be on average of high ability may choose different inputs than other parents, but they are assumed to not make input choices in a way that differentiates with respect to perceived child-specific ability.

Given that achievement is measured for each sibling at the same age, the older child's achievement observation (say child  $i$ ) will have occurred at a calendar time prior to the younger sibling's observation. Thus, the older sibling's achievement outcome was known at the time input decisions for the younger child were made, at the ages of the younger child between the older and younger child's achievement observations. Therefore, consistent estimation of (6) by ols also requires that input choices are unresponsive to prior sibling outcomes.<sup>15</sup> With regard to omitted inputs, it is necessary to assume that omitted inputs are invariant across children of the same age or are orthogonal to included inputs.

(iv) *Value added specification*

A value-added specification is often adopted when data on lagged inputs are missing or incomplete. In its most basic form, the value-added specification relates an achievement outcome measure to contemporaneous school and family input measures and to a lagged (baseline) achievement measure:

$$T_{ija} = X_{ija}\alpha + \gamma T_{ij,a-1} + e_{ija}. \quad (7)$$

To see the restrictions imposed by this form of the value-added specification, subtract  $\gamma T_{ij,a-1}$  from both sides of (2) and collect terms to get

$$\begin{aligned} T_{ija} = & X_{ija}\alpha_1 + \gamma T_{ij,a-1} + X_{ija-1}(\alpha_2 - \gamma\alpha_1) + \dots + X_{ij1}(\alpha_a - \gamma\alpha_{a-1}) \\ & + (\beta_a - \gamma\beta_{a-1})\mu_{ij0} + \{e_{ija} - \gamma e_{ij,a-1}\} \end{aligned} \quad (8)$$

where  $e_{ija} - \gamma e_{ij,a-1} = v_{ija}\rho_1 + v_{ija-1}(\rho_2 - \gamma\rho_1) + \dots + v_{ij1}(\rho_a - \gamma\rho_{a-1}) + \varepsilon_{ija} - \gamma\varepsilon_{ija-1}$ . For (8) to reduce to (7), we require<sup>16</sup>:

- (i) Coefficients associated with observed inputs geometrically (presumably) decline with distance, as measured by age, from the achievement measurement and the rate of decline is the same for each input, (i.e.  $\alpha_j = \gamma\alpha_{j-1}$  for all  $j$ ).

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<sup>15</sup>In essence, this estimation procedure can be justified when intra-household allocation decisions are made ignoring child-specific endowments and prior outcomes of all the children in the household (Rosenzweig, 1986).

<sup>16</sup>See also Boardman and Murnane (1979) for related discussion of these conditions. Equation (8) is simply the well known Koyck transformation.

- (ii) Condition (i) also holds for omitted inputs ( $\rho_j = \gamma\rho_{j-1}$  for all  $j$ ) and the contemporaneous omitted input  $v_{ija}$  is uncorrelated with included inputs and with the baseline test score; or omitted inputs (current and lagged) are uncorrelated with included inputs and with the baseline test score.
- (iii) The impact of the endowment geometrically declines at the same rate as input effects, i.e.,  $\beta_a = \gamma\beta_{a-1}$ .

For the ols estimator of  $\alpha_1$  to be consistent,  $\varepsilon_{ija}$  must also be serially correlated, with the degree of serial correlation matching the rate of decay of input effects (so that  $\varepsilon_{ija} - \gamma\varepsilon_{ija-1}$  is an iid shock). If this condition is not satisfied, then baseline achievement,  $T_{ija-1}$ , will be correlated with its own measurement error.

(v) *Value-added plus specification*

When data are available on historical input measures, then assumption (i) can be relaxed. A value-added specification that does not impose (i) would include as additional regressors data on lagged inputs.

As noted in the introduction, recent work by Cuhna, Heckman, Lochner and Masterov (2006) and Cuhna and Heckman (2003) adopt a value-added to jointly model the formation of cognitive and noncognitive skills. In terms of our notation, their specification could be described as:

$$\begin{pmatrix} T_{ijt+1}^N \\ T_{ijt+1}^C \end{pmatrix} = A_t \begin{pmatrix} T_{ijt}^N \\ T_{ijt}^C \end{pmatrix} + B_t X_{ija} + \begin{pmatrix} e_{ijt}^N \\ e_{ijt}^C \end{pmatrix},$$

where  $T_{ijt+1}^N$  denotes a measure of noncognitive skills and  $T_{ijt+1}^C$  a measure of cognitive skills and  $X_{ija}$  are current inputs. If one were to focus only on the cognitive test score measure equation and to substitute repeatedly for the right-hand-side  $T_{ijt}^N$  variables, then a value-added plus specification would be obtained. In that sense, the model they implement is consistent with the value-added plus specification.

### 3 Data

As described in section two, the data requirements for implementing the cumulative specifications of the cognitive achievement production function are demanding. A researcher needs a complete history of inputs, beginning at the child's conception. In addition, to account for unobserved endowments one needs multiple observations on achievement measures, either for siblings at the same ages or for the same child at different ages. Although there does not exist a data set that satisfies all these requirements, the NLSY79 Child Sample (NLSY79-CS) comes close to meeting them.

The NLSY79-CS is a sample of all children ever born to the women respondents of the NLSY79. The NLSY79 is itself a nationally representative sample of individuals who were age 14-21 as of January 1, 1979, with significant oversamples of blacks and Hispanics. The survey collects extensive information about schooling, employment, marriage, fertility, income, assets, alcohol and drug use, participation in public programs and other related topics, many as event histories. For example, employment events are known up to the week, marriage and fertility events to the day and school enrollment to the month. This enables the researcher to create an almost complete life history for each respondent for many important events dating back to age 14.

Beginning with the 1986 interview, a separate set of questionnaires were developed to collect information about the cognitive, social and behavioral development of the children of the NLSY79 respondents. Questionnaires were administered to the women (cum mothers) of the children as well as to the children themselves. These interviews have been conducted biannually since 1986. By 2000, over 11,000 children were interviewed. Approximately 28 percent of the children in 2000 were African American, 19 percent Hispanic and the rest mostly white.

**Cognitive Achievement Measures** Our analysis restricts attention to two cognitive tests that were administered to all children starting at age five: the Peabody Individual Achievement Test in mathematics (PIAT-M) and the Peabody Individual Achievement Test in reading recognition (PIAT-R). The PIAT tests are designed to measure academic achievement. They were administered each year of the survey, and many of the children in the sample have two or more scores. Completion rates for the PIAT's have been around 90 percent.

Table 1 shows the average PIAT Math and Reading scores by race/ethnicity. In our work, we use both raw test scores and age-normed percentile scores. The raw score provides an absolute measure of achievement that captures gains in knowledge over time as additional input investments are made in a child, while the percentile score is a relative measure of performance, useful for making group comparisons.

Figures 1a and 1b plot the average PIAT Reading and Math percentile test scores by age, by gender and by race/ethnicity.<sup>17</sup> The lower panel shows the black-white and Hispanic-white gap. At age six, the gap in reading scores for both black and Hispanic children relative to white children is 10 percentile points. The gap remains roughly constant with age for

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<sup>17</sup>The survey is a biennial survey, so children are typically interviewed at even ages 6,8,10,12.or at odd ages 7,9,11,13. For this reason, we report in the figure averages over age categories 6-7,8-9,10-11,12-13.

Hispanic children, but widens for black children, particularly for black boys over age 6-9. By age 12-13, the gap for both black boys and girls is approximately 20 percentile points. For math test scores, there is also a sizable gap at age 6 between whites and minorities. As with reading, the black-white test score gap for boys widens markedly over ages 6-9. For black girls, the gap continues to grow through age 12, but more gradually. The white-Hispanic gap in the reading score exhibits some widening, then convergence for boys and some convergence and then widening for girls. By age 12-13, the math gap for black boys is much greater than for Hispanic boys, whereas the gaps for black and Hispanic girls are similar.

**Home Input Measures** The NLSY79-CS includes a battery of questions about the home environment of the child called the Home Observation Measurement of the Environment-Short Form (HOME-SF).<sup>18</sup> The HOME-SF consists of four different instruments that depend on the age of the child: ages 0-2, 3-5, 6-9 and 10 and above. The instrument is (self-administered) to the mother of the child. A second version is filled out by the interviewer. Researchers can use either the answers to individual items or scales provided in the public use files. The total raw score is a simple summation of responses (modified so each has a {0,1} domain}) of individual items.

Some of the items in the home can be directly linked to cognitive achievement in the sense that they are related to learning-specific skills. For example, mothers of children under the age of 10 are asked how often they read stories to their child, and mothers of children between the ages of 3 and 5 are asked whether they help their child to learn numbers, the alphabet, colors or shapes and sizes. Other items are not so easily tied to cognitive achievement, but may be thought of as creating an environment conducive to learning. For example, mothers are asked how many books the child has, whether the family encourages the child to start and keep doing hobbies, and whether the family takes the child to museums and/or theatrical performances.

In the empirical work reported below, we use the home scale provided in the public use files as our measure of the time and goods inputs provided in the home. As described in section two, we consider both current home inputs and historical home inputs as potential determinants of current test scores. To get a better idea of what the home scale measures, Tables A.1-A.4 in appendix A compare the average scores by race/ethnicity for the individual

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<sup>18</sup>As the name suggests, the short form is a modification of a version that is about twice as long. The HOME was created by Caldwell and Bradley (1984). Some parts of the shortened version used in the NLSY79-CS were created by them and all were reviewed by them. The HOME (-SF) is widely used and there exists considerable research on the validity and reliability (see the citations in the 1996 *Users Guide*).

items of the cognitive home scale for children in different age ranges.<sup>19</sup> About 2/3 of the items in the home scale are based on mother self-reports of her own and her child’s activities and about 1/3 of the items correspond to interviewer observations about the child’s home environment. The average scores for the African American and Hispanic mothers tend to be similar and tend to be lower than the scores for white mothers for most of the individual items. The differences are particularly notable for the questions related to number of books in the child’s possession, the number of times the mother reads to the child, and the teaching activities the mother engages in with the child. For example, 94% of white mothers report that their age 3-5 toddler has 10 or more books in comparison with 57% of black mothers and 63% of Hispanic mothers. The difference in book ownership persists for children in all the age ranges. 70% of these same mothers report reading stories to their toddler at least 3 times a week, in comparison with 40% of black mothers and 44% of Hispanic mothers. 66% of black mothers and 70% of Hispanic mothers report teaching their age 3-5 child numbers in comparison with 78% of white mothers. For older children age 6-9, 61% of white children receive special lessons or participate in organizations that encourage sports, arts, dance or drama, compared to 41% for black children and 39% for Hispanic children. The items of the home scale based on interviewer observations also show some differences by race/ethnicity, but they tend to be smaller than the differences observed on the self-report items. Thus, examination of the individual items of the home scores reveals some stark racial/ethnic differences for children in all the age ranges, especially for the items that are self-reported by the mother related to books, reading and teaching activities.

Table 1 shows the average home score for our analysis samples, for ages 6-13 and for ages 3-5.<sup>20</sup> The average white home score at ages 3-5 is 13% higher than the average black home score, and 12% higher than the average Hispanic home score. At ages 6-13, the white score is 15% higher than the black score and 13% higher than the Hispanic score.

Figure 2 plots the current home score by age, gender and race/ethnicity. The plots show that the gap in home scores (relative to whites) is similar for blacks and Hispanics, and declines somewhat with age. The plots by gender show that black boys have slightly lower home scores than Hispanic boys, but the reverse is true for girls.

**Maternal Characteristics** Information is available on mother’s completed schooling, which is updated in each year in which the mother attended school. Because some women return to school after having children, both within-family and within-child estimators can be

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<sup>19</sup>The questions that are asked of the mother differ slightly across four different age ranges.

<sup>20</sup>We distinguish these age groups because the questions that comprise the home score scale are substantially different for 3-5 year-olds.

used to estimate maternal schooling effects on children’s achievement.<sup>21</sup> Women with higher school attainment presumably have more knowledge to transmit to their children and/or may be better teachers. A comparison of mothers’ schooling levels by race/ethnicity shows that white mothers have the highest average years of schooling (13.1), African American mothers the second highest (12.4), and Hispanics the lowest (11.7).

In addition to schooling, the NLSY79 also contains a measure of ability for the mothers, their score on the Armed Forces Qualifying Test (AFQT). A direct measure of mother’s knowledge is a potentially important factor in the production of cognitive skills in children. As seen in Table 1, the AFQT score for white mothers is close to the median (52.4), while the average percentile rank for African American and Hispanic mothers, 20.4 and 25.6, is much lower.

The NLSY79 includes only limited information about fathers. In fact, identifying the biological father is problematic. Although the public-use data include a variable indicating presence of the biological father in the household, the variable is missing in many cases.

**Child Characteristics** In addition to standard information on race and gender of the child, the NLSY79-CS also contains information on other characteristics that are potential determinants of a child’s cognitive achievement, such as birth order and birthweight.<sup>22</sup> As shown in Table 1, African American children have on average lower birthweight than white or Hispanic children. The disparity of about 6-8 ounces is due either to biological factors or to differences in prenatal investments. White children are more likely to be first or second born because white women have fewer children.

**School Inputs** The major weakness of the NLSY79-CS is the paucity of data on schools. Implementing the cumulative model described by equation (1) in the previous section requires both contemporaneous and historical data on school inputs. We therefore obtain schooling data from other sources that we merge with the NLSY79-CS data using information on the child’s grade, county and state of residence, and whether the child was attending private school.<sup>23</sup>

In particular, we use the Common Core Data (CCD), which contains information on all public schools and on the characteristics of students both at the school and district levels. In

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<sup>21</sup>Rosenzweig and Wolpin (1994) exploit the interruption in schooling that occurred for some NLSY79 mothers.

<sup>22</sup>See, e.g., Rosenzweig (1986).

<sup>23</sup>County and state of residence are available at each survey round of the NLSY79 respondents (and their children) and can be obtained as a restricted data file from the Bureau of Labor Statistics.

the CCD, schools report the number of full-time equivalent teachers and the number of pupils enrolled, which we use to calculate pupil-teacher ratios for each school. Because elementary grades and upper level grades are usually offered in separate schools, we obtain separate pupil-teacher ratio averages for grades 1-6 and grades 7-12. We constructed both county level and state level pupil-teacher ratio variables, which we merged with the NLSY79-CS data. We also obtained a series of average teacher salaries by state, for the years 1984-2001, from the American Federation of Teachers.

The schooling inputs on which we focus in the analysis are pupil-teacher ratios and teacher salaries. We also estimated specifications using data on teacher's education, teacher experience, hours/week spent teaching math and English (separately), and teacher certification. These variables do not appear in the final specifications as inputs, because estimates of their effects were never precise. We therefore adopted a more parsimonious specification that includes two conventional measures of school inputs: pupil-teacher ratios and teacher salaries.

Table 1 shows average pupil teacher ratios and average teachers' salaries for white, African American and Hispanic children, where the average is taken over the child's school history for the years in which the school input measures are available.<sup>24</sup> Although historically, African American children attended schools that were of much lower quality than white children, there has been substantial convergence in empirical measures of schooling quality over time. Boozer, Krueger, and Wolken (1992) note that in 1970 the pupil-teacher ratios in schools attended by black children were on average 11% higher than in schools attended by whites, but by 1990 there was no difference.<sup>25</sup> In our schooling data, the average pupil-teacher ratios are lowest for African American children (18.1) and highest for Hispanic children (19.8). Teacher salaries on average are highest for Hispanic children (\$32,218) and lowest for African American children (\$29,624).

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<sup>24</sup>We attempted to construct separate contemporaneous and lagged average measures of school inputs (as with the home inputs), but, perhaps due to the higher level of aggregation, there was substantial colinearity and we were unable to obtain precise estimates of their separate effects. Therefore, we use one cumulative measure.

<sup>25</sup>See also Card and Krueger (1992) for evidence on the convergence of schooling quality in black and white schools over the last century and an analysis of the effects of convergence on earnings. Donohue, Heckman and Todd (2002) study the sources of convergence in the South over the 1911-1960 time period.



## 4 Empirical Results

As described in section two, a benefit of the rich longitudinal data is that they enable estimation of more general specifications that accommodate the presence of unobserved endowments and input choices that are endogenous with respect to those endowments. Here, we estimate all of the specifications described in the section two: the contemporaneous specification, the cumulative specification with orthogonal endowments and unobserved inputs, within child and sibling fixed-effect specifications, a value-added specification, and the value-added plus specification (described in section two, which includes lagged inputs). As previously noted, these specifications impose different sets of restrictions on the most general model, given in (2).

### 4.1 Estimating Equations

The cumulative model presented in section two relates test scores to current and lagged home and school inputs. Children are interviewed approximately every two years, so we assume that the inputs apply to a two-year interval. Table 2 summarizes the current and lagged inputs that are available for children of different ages. We specify the dependence of test scores on lagged home and school inputs as follows:

$$\begin{aligned}
 T_a = & \alpha_0 Home_a + \alpha_{11} LagHome_a^{-1} I(age\ 6-7) + \alpha_{12} LagHome_a^{-1} I(age\ 8-13) + \quad (9) \\
 & \alpha_{21} LagHome_a^{-2} I(age\ 8-9) + \alpha_{22} LagHome_a^{-2} I(age\ 10-13) + \\
 & \alpha_{31} LagHome_a^{-3} I(age\ 10-11) + \alpha_{32} LagHome_a^{-3} I(age\ 10-13) + \\
 & \alpha_4 LagHome_a^{-4} I(age\ 12-13) \\
 & + \delta_1 PTRAvg_a \times S_a + \delta_2 TchSalAvg_a \times S_a + \varphi AFQT + \gamma X_a + \beta_a \mu + \varepsilon_a,
 \end{aligned}$$

where  $T_a$  represents the test score at age  $a$ .  $Home_a$  represents the contemporaneous home input and  $LagHome_a^{-k}$  represents the  $k$ th period lagged home input. The coefficient on the home input measure is allowed to differ when the lag corresponds to the measured input at age 3-5, because the battery of questions that comprise the home scale in those years is different (see section three). The variables  $PTRAvg_a$  and  $TchSalAvg_a$  are the average of the school quality variables (pupil-teacher ratios and teacher salary) taken over the years in which the child attended school. This average quality is multiplied by the number of years attended ( $S_a$ ) to get the cumulative effect of having been exposed to that level of school quality. Although the school quality variables could have been treated symmetrically with the home input variables, by including separate lags instead of the average, this was

impractical because of colinearity in the lag-specific school measures.<sup>26</sup> We also include in the specification the mother’s AFQT test score measure under the presumption that mothers with a higher set of skills have a technological advantage in the production of children’s achievement. Lastly, the variable  $X_a$  represents additional covariates in the specification that include birth weight, indicators for whether child is the first or second born child, indicators for mothers age at the time of birth, the child’s age in month and its square as well as separate indicator variables for each age, and the mother’s schooling level.<sup>27</sup>

As noted in section two, one of the major difficulties in estimating the cognitive achievement production function is how to account for missing data on inputs. One approach to this problem that may be less than satisfactory is to assume that missing inputs are orthogonal to included inputs and therefore do not bias the estimation of their associated coefficients. Another option is to specify input demand equations that express the missing inputs as functions of family income, prices and preference shocks and to substitute the input demand equation in place of the missing input in the production function. We illustrate the application of this second approach, which we adopt in the empirical work, with a simple utility maximizing model. Assume that parents (with one child) maximize utility that depends on consumption and child achievement. They can purchase three different inputs,  $X_1$ ,  $X_2$ , and  $X_3$  to produce achievement at prices  $p_1$ ,  $p_2$  and  $p_3$ . Assume that data are available on the first two inputs, but are missing on the third. The problem solved is:

$$\begin{aligned} & \max_{X_1, X_2, X_3} U(A(X_1, X_2, X_3, \varepsilon), C, \eta) \\ & s.t. \\ & C + p_1 X_1 + p_2 X_2 + p_3 X_3 = y, \end{aligned}$$

where  $A$  is the child achievement production function,  $y$  represents family income,  $\varepsilon$  is a shock to the achievement production function, and  $\eta$  is a preference shock (e.g. a shock to the marginal utility from achievement). The price of consumption has been normalized to one. Assuming linear input demand equations gives the input as a function of prices, family income and preference and technology shocks (contained in  $\nu$ ):

$$X_j = \gamma_0^j + \gamma_1^j p_1 + \gamma_2^j p_2 + \gamma_3^j p_3 + \gamma_4^j y + \nu_j.$$

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<sup>26</sup>The school quality measures are state-level averages, measured separately for primary, middle and secondary school. The averages are constructed in a way that takes into account state-to-state migration by children.

<sup>27</sup>The specification does not include presence of father in the household, because in many cases that variable is missing. When we did include it and estimated the equation for the subsample for which it is available, its coefficient was imprecisely estimated.

To address the problem of estimating the parameters of the achievement production function when data on  $X_3$  are missing, we substitute the input demand equation to obtain a hybrid specification:

$$A = \tilde{\alpha}_0 + \tilde{\alpha}_1 X_1 + \tilde{\alpha}_2 X_2 + \tilde{\alpha}_3 \gamma_0^j + \tilde{\alpha}_3 \gamma_1^j p_1 + \tilde{\alpha}_3 \gamma_2^j p_2 + \tilde{\alpha}_3 \gamma_3^j p_3 + \tilde{\alpha}_3 \gamma_4^j y + \{\tilde{\alpha}_3 \nu_j + \varepsilon\},$$

where the terms in brackets comprise the residual.<sup>28</sup> If prices are constant across all the observations, then their effect would be absorbed into the model intercept. This hybrid specification now explicitly accounts for the missing input  $X_3$ . However, applying ols to the estimation of the hybrid specification is problematic because the shocks in the input demand equations are likely to be correlated ( $E(\nu_j \nu_k) \neq 0$ ), implying a non-zero correlation between the observed included inputs and the error term. There is no a priori reason to expect that the bias in estimating the technology parameters  $\tilde{\alpha}_1$  and  $\tilde{\alpha}_2$  would be smaller under the hybrid specification than under the original specification that includes  $X_1$  and  $X_2$  and simply omits  $X_3$ .

In the empirical work, we consider specifications of the form (9) as well as a hybrid specification that adds to the list of covariates indicators for race, sex, and a variable capturing cumulative family income over the child's lifetime.<sup>29</sup> Our motivation for including race and gender in the hybrid model is to allow family preferences for achievement to potentially vary by race and child gender.

## 4.2 Estimated Production Function Coefficients

Tables 3a and 3b report the estimated coefficients for alternative specifications described in section two of the paper for PIAT math and reading percentile test score measures. Tables 4a and 4b report analogous results for the "hybrid" specifications. Tables A.5a, A.5b, A.6a, A.6b, included in the appendix, report the estimated coefficients when the raw test score measure is used instead of the percentile measure.

As discussed in section three, the contemporaneous specification places strong restrictions on the production technology but is less demanding than other specifications in terms of data requirements. Under the null that the contemporaneous model is correctly specified, test scores are a function only of current input measures. The first column of Table 3a, labeled

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<sup>28</sup>See Rosenzweig and Schultz (1983) for an explicit interpretation of hybrid production function parameters.

<sup>29</sup>The same approach applied to a cumulative specification would substitute the input demand equations for both contemporaneous and lagged home inputs. These would depend on current and lagged family income.

"Contemporaneous," presents results for the math test score and for the specification that includes only current home inputs. The current home inputs are statistically significantly different from zero as is the mother's AFQT score. The corresponding estimates for the reading percentile score, shown in Table 3b, exhibit the same patterns. The coefficients on the pupil-teacher ratio and on teacher salary are of the expected signs. The teacher salary measure is statistically significant at a 10% level for both the math and reading percentile scores, but the pupil-teacher ratio is only statistically significant for the math percentile score.<sup>30</sup> The magnitude of the implied effects of schooling quality on test scores is not very large. The estimates imply, for example, that a change in the average pupil-teacher ratio by five fewer students would lead to an increase in the math percentile score of 1.8 (for a child with three years of schooling) and an increase of 1.2 in the math percentile score. A \$10,000 increase in teacher salary (in 1989 dollars) would lead to an increase of 2 in the math percentile score and 1.8 in the reading percentile.

A straightforward test of the contemporaneous specification that is implementable when historical data on inputs are available, is to include the historical input measures and check whether their associated coefficients are significantly different from zero. The second column of tables augments the contemporaneous specification with lagged data on home inputs. As described above, we allow the coefficients associated with lagged home inputs measured at ages 3-5 to differ because the questions in the home scale were different at that age. The estimated effect of the current home input falls to less than half its original magnitude when lagged inputs are included. Thus, omitting historical measures leads to an overstatement of the impact of a unit increase in Current Home input. Also, neglecting the influence of historical measures understates the impact of a unit increase in the home score that is sustained over an extended time period. For example, the estimates for the reading specification (Table 3b, column (2)) imply that a unit increase in the home score at ages 3-5, 6-7 and 8-9 increases the PIAT Reading test score at age 10-11 by  $0.068+0.061+0.088=0.217$ , which is thirty-eight percent larger than the effect implied by the contemporaneous specification (0.157). The estimated coefficients on all but one of the lagged inputs (the third lag at age 12-13) are positive and statistically significantly different from zero, which is evidence against the contemporaneous specification. Including the lagged home input measures, however, leads to a decline in the statistical significance of the pupil-teacher ratio, particularly for the

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<sup>30</sup>A school input that we cannot measure is the curriculum content within the classroom. A proxy for curriculum could be the grade level the child is currently attending. However, to the extent that grade progression depends on prior achievement, grade level would reflect all past inputs and would be inappropriate to include. Also, if grade progression were automatic, age effects included in our specification would capture grade-specific curriculum content.

reading score.

The third column of Table 3a implements the value-added specification including only contemporaneous home inputs. The coefficient on the home input variable and on the lagged test score measures are statistically significantly different from zero and of the expected sign (positive). Teacher salary is only statistically significant for the reading score and pupil-teacher ratio only for the math score. The fourth column of the table presents the estimates for the value-added plus model, which adds to the basic value-added lagged data on inputs. Only two lagged inputs are included, because the additional lags were never precisely estimated. For both the reading and math scores, the additional lagged inputs are individually and jointly statistically different from zero. The magnitude of the coefficients on the school quality variables is virtually unchanged by adding the additional lagged inputs.

The fifth and sixth columns of Table 3(a) present estimates for the within family and within child fixed effect estimators. In these models, we cannot estimate the effect of mother's AFQT, because it does not vary with age of the child or across children. The contemporaneous home input and most of the lagged input variables continue to be statistically significant determinants of test scores in both the sibling fixed-effect and the child fixed-effect specifications. Some of the coefficients associated with lagged inputs vary depending on whether the within-family or the within-child estimator is implemented. The school quality measures are generally not significant in any of the fixed effect specifications, with the exception of the pupil-teacher ratio for the math score.

In addition to the results reported in the tables, we also estimated specifications where the pupil-teacher ratio was measured at the county level rather than the state level.<sup>31</sup> The county level measure was less often statistically significant for all the specifications, which is consistent with other findings reported in the literature. For example, Card and Krueger (1996) found significant effects of state-level school quality measures on earnings. Betts (1995) compares the estimated effect of pupil-teacher ratios and teacher experience on earnings under different levels of aggregation of the quality measures and finds that the quality measures are only significant when measured at the state level.<sup>32</sup>

Tables 4a and 4b show the analogous estimated coefficients for the hybrid production function models that include as additional covariates cumulative family income, race and

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<sup>31</sup>County is the most detailed measure of location available for the NLSY79 respondents.

<sup>32</sup>Our state-level measure of quality differs in some ways from measures in the literature, where state often corresponds to a person's state of birth and it is assumed that the child is educated in their state of birth. In our case, the state measure gives the state level average quality at the time of the child's residence. If a child moves from one state to another, our average school input measures would change to reflect different levels of school inputs across states and to reflect the amount of time spent in each location.

gender to account for missing data on inputs. The estimates of the production function parameters change somewhat when these additional variables are included. The effect of these changes on minority-white differences in test scores are discussed below.

### 4.3 Within Sample Goodness of Fit

As seen in Tables 3a,b and 4a,b, inferences about the contribution of home and school inputs in explaining test scores depend to some extent on the model specification. We next visually assess the performance of the alternative models in reproducing the test score gap patterns and how the gaps patterns vary by age, race/ethnicity, and gender. Figures 3a-d and 4a-d present the predicted gap pattern superimposed on the actual gap pattern for the alternative specifications, both for the non-hybrid and hybrid versions of the production function and for the math and reading percentile scores, by race/ethnicity and gender. The hybrid model includes race, gender, and cumulative family income as additional covariates, which considerably improves the fit to the gap patterns. The fixed effect models are included as variants of hybrid models, because the fixed effects implicitly include race/ethnicity and gender. As seen in the figures, the contemporaneous specification usually does not reproduce the gap pattern, either for boys or for girls. In particular, it fails to generate the observed widening gap for both black girls and boys. The cumulative models without endowments is able to better reproduce the rising gap pattern, but it generally does not fit the gap pattern as well as the value-added, value-added plus and fixed effect models. There are some subgroups (see Figures 4c-4d) for which the value-added plus model is clearly a better fit than the fixed effects models.

### 4.4 Model Selection

As noted in section one, one of our goals in estimating the production function is to decompose the observed racial/ethnic test score gaps into components due to home, school and mother's AFQT differences. When we give the minority groups the average home input levels observed for whites, we are in essence using the model to perform an out-of-sample forecast. We explore the reliability of the estimated models for out-of-sample forecasting purposes using a cross-validation methods. These methods compare models on the basis of an out-of-sample root-mean-squared-error criterion (RMSE). They can be used to compare non-nested models. They cannot, however, be used to determine whether the hybrid or non-hybrid versions of the production function model exhibit less bias in the estimated input effects, because they do not speak directly to the bias of a subset of the production

function parameters.

We performed the cross-validation in two ways, using random hold-out samples (the conventional approach) and using selective hold-out samples that correspond to groups based on race and gender. The cross-validation procedure based on random hold-out samples is implemented as follows. First, the entire sample is randomly divided into six roughly equal-sized subsamples. The model is repeatedly estimated on five of the six subsamples and used to construct the RMSE for the left-out subsample, alternating which group is left out. The RMSE values for each subsample are then summed to obtain the overall RMSE for that model. We constructed the overall RMSE for three different initial randomizations and report in Table ? the CV-RMSE for each specification. We also performed the cross-validation exercise for nonrandom hold-out samples, leaving out one race $\times$ gender group at a time. As seen in the table, under either the random hold-out sample criterion or the selective hold-out sample, the specification that repeatedly exhibits the lowest CV-RMSE is the value-added model, with the additional lagged inputs, and in one case without lagged inputs.

## 5 Accounting for Sources of Racial Test Score Gaps

Using the production function estimates from the last section, we examine the extent to which differences in inputs and in mother's AFQT can account for racial/ethnic disparities in test scores. The implied impacts of our schooling quality measures on test scores were very small in comparison to those of the home inputs and AFQT, so we do not include them in the decomposition.<sup>33</sup> The decomposition results are presented for the value-added plus model, which was the preferred model according to the cross-validation criterion.

Table 5a examines how eliminating the gap in home inputs and in mother's AFQT would close the gap in test scores. That is, we assess the extent of the gap if black average home inputs were set to the level observed for white children and if black mothers' AFQT scores were set to the average level of white children. The first column of the table shows the Actual test score Gap in the raw scores and in the percentile scores, averaged over all ages (6-13). The second and third columns (labeled "Prediced Gap") give the predicted gap according to the non-hybrid and hybrid specifications of the production function. The fourth and fifth columns (labeled "Closed by Home") give the amount of the predicted gap that would be closed if black children received on average the level of white home inputs. Roughly one quarter of the test score gap in math and reading would be closed by equalizing

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<sup>33</sup>Given AFQT, mother's schooling also accounts for little of the gap.

home inputs. The last two columns (labeled "Closed by AFQT") show the amount of the gap that is closed by equalizing mother's AFQT. The estimates indicate that roughly half of the predicted gap is accounted for by differences in mothers' AFQT. Table 5b presents analogous results for the white-Hispanic decomposition. The findings concerning the relative contribution of mother's AFQT and home inputs are generally similar to those of the black-white decomposition. One difference is that home inputs explain a larger proportion of the predicted reading gap for Hispanics than for blacks, roughly 35%.

To summarize, the decomposition estimates indicate that racial/ethnic differences in mother's AFQT generally explains a larger fraction of the gap in test scores than do differences in home inputs. However, the contribution of home inputs is on the order of 25-35%, which is not negligible. Our estimates thus imply that equalizing home inputs of whites and blacks, holding all other inputs constant, would close a significant proportion of the test score gap. To the extent that home inputs affects AFQT scores (as it does the PIAT scores), this would also have intergenerational benefits.

## 6 Conclusions

This paper considered ways of estimating the cognitive achievement production function that are consistent with theoretical notions that achievement is a cumulative process depending potentially on the entire history of family and school inputs as well as on parental ability and unobserved endowments. Using rich longitudinal data, we implemented alternative specifications of the production function. Across almost all the specifications considered, we found that mother's ability and home inputs (contemporaneous and lagged) are substantively significant determinants of child test scores. The magnitude of lagged home input effects is often similar to that of current inputs. The coefficients associated with school inputs (pupil-teacher ratio and teacher salaries) were only found to be significant determinants of test scores in specifications that did not allow for fixed effects.<sup>34</sup>

When alternative models were compared using a cross-validation criterion, we found the most support for the "Value-added plus" model, which augments a basic value-added model with additional lagged input variables. We used the production function parameter estimates for this preferred specification to examine the sources of racial/ethnic test score gaps. Differences in mother's AFQT account for the largest portion of the black-white and Hispanic-white test score gaps—roughly half for both reading and math. Differences in home

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<sup>34</sup>To estimate the effects of school inputs more precisely, it be desirable to have school inputs measured at the same level of aggregation as the home input measures (i.e. classroom level instead of state level).



inputs account for 25% of the black–white test score gap and about 30% of the Hispanic–white gap. Differences in school inputs and in mother’s schooling account for only very small portions of the gap.

Our findings do not imply that the most efficient way to close the gap is to invest in ways of augmenting home inputs. What is required to make such determination is knowledge of the relative costs of alternative policies and of how schools and parents make input decisions, to account for the possibility that changing the level of a single input affects decisions about other inputs. A full assessment of such policies would require a complete analysis of how families make decisions about what inputs to provide for their children.<sup>35</sup>

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<sup>35</sup>For example, see recent efforts by Mroz, Liu and Van der Klaauw (2003) and Bernal and Keane (2005).

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Table 1  
Descriptive Statistics: Means,  
Standard Deviations in Parentheses

	White	Black	Hispanic
Piat Math (Age 6-13) - Percentile	61.9 (24.5)	42.4 (25.2)	47.2 (26.4)
Piat Reading (Age 6-13) - Percentile	60.1 (26.5)	44.2 (27.2)	48.9 (28.9)
Piat Math (Age 6-13) - Raw	42.0 (14.2)	34.9 (13.7)	36.0 (14.3)
Piat Reading (Age 6-13) - Raw	40.5 (14.0)	34.5 (12.4)	36.0 (14.0)
Current Home Score (Age 6-13)	104.8 (20.6)	88.9 (23.8)	91.3 (24.5)
Average Home Score Age 3-5	123.2 (16.9)	106.0 (23.3)	108.3 (25.6)
Average (over all of the child's school years) Pupil-Teacher Ratio	18.3 (2.4)	18.1 (2.4)	19.8 (3.3)
Average (over the child's school years) Teacher Salary (1989 \$)	31,661	29,624	32,218
Child Age (in months)	113.8 (24.6)	113.4 (24.9)	111.9 (24.4)
Birth weight (ounces)	120.8 (20.2)	112.0 (21.7)	118.9 (20.7)
Percent Firstborn	48.3	33.9	40.3
Percent Second born	35.8	37.8	34.5
Percent of Children with Mother's age at birth			
Less than 18	1.1	1.5	1.7
18-19	5.0	10.9	11.0
20-29	82.0	80.7	80.1
Mother's Schooling	13.1 (2.3)	12.4 (2.0)	11.7 (11.7)
Mother's AFQT Percentile Score	52.4 (26.0)	20.4 (17.7)	25.6 (22.1)
Number of Observations	3,802	2,403	1,495

Table 2  
Lags Home Input Measures for Test Score Equation at Different Ages  
and Corresponding Regression Coefficients

Age at Test Score Measure	Age at Current home	Age at One Period Lagged Home	Age at Two Period Lagged Home	Age at Three Period Lagged Home	Age at Four Period Lagged Home
6-7	6-7 ( $\alpha_0$ )	3-5 ( $\alpha_{11}$ )	...	...	...
8-9	7-8 ( $\alpha_0$ )	6-7 ( $\alpha_{12}$ )	3-5 ( $\alpha_{21}$ )	...	...
10-11	10-11 ( $\alpha_0$ )	8-9 ( $\alpha_{12}$ )	6-7 ( $\alpha_{22}$ )	3-5 ( $\alpha_{31}$ )	...
12-13	12-13 ( $\alpha_0$ )	10-11 ( $\alpha_{12}$ )	8-9 ( $\alpha_0$ )	6-7 ( $\alpha_{32}$ )	3-5 ( $\alpha_4$ )



Table 3a: Alternative Specifications of the Cognitive Achievement Production Function: Math Percentile Score<sup>a</sup>  
(robust standard errors in parentheses)

	Contemporaneous	Cumulative	Value Added		Sibling FE	Child FE
			(1)	(2)		
<b>Home Inputs</b>						
(1) Home (0)	.154 (.017)	.059 (.016)	.089 (.014)	.054 (.017)	.069 (.017)	.053 (.018)
(2) Home (-1)						
Age 6-7	-	.163 (.024)	-	-	-.0001 (.031)	.040 (.030)
Age 8-13	-	.069 (.018)	-	.018 (.018)	.054 (.018)	.038 (.018)
(3) Home (-2)						
Age 8-9	-	.208 (.026)	-	.100 (.022)	.071 (.028)	.089 (.024)
Age 10-13	-	.073 (.022)	-	.036 (.018)	.063 (.022)	.052 (.021)
(4) Home (-3)						
Age 10-11	-	.134 (.034)	-	-	.017 (.030)	-
Age 12-13	-	-.006 (.032)	-	-	.004 (.029)	-
(5) Home (-4)						
Age 12-13	-	.121 (.040)	-	-	-	-
Lag Test Score	-	-	.568 (.015)	.560 (.015)	-	-
Mother's AFQT	.342 (.021)	.307 (.021)	.171 (.016)	.162 (.016)	-	-
Pupil-Teacher Ratio	-.122 (.050)	-.103 (.049)	-.070 (.029)	-.069 (.030)	-.087 (.046)	-.129 (.046)
Teacher Salary	6.67E-5 (2.8E-5)	4.99E-5 (2.8E-5)	2.86E-5 (1.7E-5)	2.74E-5 (1.73E-5)	1.13E-5 (2.8E-5)	3.71E-5 (2.73E-5)

a. Also includes birth weight, first and second born dummies, dummies for mother's age at birth 18-19 and 20-29, child's age in months and its square, dummies for child age in years, mother's schooling.

Table 3b: Alternative Specifications of the Cognitive Achievement Production Function: Reading Percentile Score<sup>a</sup>  
(robust standard errors in parentheses)

	Contemporaneous	Cumulative	Value Added		Sibling FE	Child FE
			(1)	(2)		
<b>Home Inputs</b>						
(1) Home (0)	.157 (.018)	.074 (.016)	.105 (.015)	.059 (.017)	.093 (.018)	.032 (.018)
(2) Home (-1)						
Age 6-7	-	.135 (.027)	-	-	.011 (.031)	-.007 (.030)
Age 8-13	-	.068 (.018)	-	.021 (.017)	.058 (.019)	.029 (.018)
(3) Home (-2)						
Age 8-9	-	.170 (.027)	-	.124 (.024)	.076 (.029)	.055 (.024)
Age 10-13	-	.061 (.021)	-	.054 (.018)	.066 (.022)	.036 (.021)
(4) Home (-3)						
Age 10-11	-	.088 (.032)	-	-	.001 (.031)	-
Age 12-13	-	-.007 (.032)	-	-	.039 (.030)	-
(5) Home (-4)						
Age 12-13	-	.125 (.037)	-	-	-	-
Lag Test Score	-	-	.453 (.013)	.448 (.013)	-	-
Mother's AFQT	.313 (.020)	.283 (.021)	.214 (.017)	.201 (.017)	-	-
Pupil-Teacher Ratio	-.085 (.049)	-.069 (.048)	-.032 (.030)	-.029 (.030)	.057 (.048)	.042 (.047)
Teacher Salary	5.98E-5 (2.7E-5)	4.44E-5 (2.6E-5)	3.42E-5 (1.7E-5)	3.20E-5 (1.7E-5)	1.00E-5 (2.9E-5)	4.15E-5 (2.8E-5)

a. Also includes birth weight, first and second born dummies, dummies for mother's age at birth 18-19 and 20-29, child's age in months and its square, dummies for child age in years, mother's schooling.

Table 4a: Alternative Specifications of the “Hybrid” Production Function: Math Percentile Score<sup>a</sup>  
(robust standard errors in parentheses)

	Contemporaneous	Cumulative	Value Added		Sibling FE	Child FE
			(1)	(2)		
<b>Home Inputs</b>						
(1) Home (0)	.124 (.017)	.048 (.016)	.078 (.015)	.051 (.017)	.069 (.018)	.048 (.018)
(2) Home (-1)						
Age 6-7	-	.142 (.024)	-	-	.002 (.031)	.056 (.030)
Age 8-13	-	.060 (.018)	-	.013 (.018)	.053 (.018)	.032 (.018)
(3) Home (-2)						
Age 8-9	-	.191 (.026)	-	.092 (.022)	.073 (.028)	.097 (.025)
Age 10-13	-	.068 (.022)	-	.028 (.018)	.062 (.022)	.044 (.021)
(4) Home (-3)						
Age 10-11	-	.118 (.034)	-	-	.018 (.030)	-
Age 12-13	-	-.014 (.032)	-	-	-.0004 (.029)	-
(5) Home (-4)						
Age 12-13	-	.107 (.039)	-	-	-	-
Lag Test Score	-	-	.559 (.015)	.553 (.015)	-	-
Mother’s AFQT	.254 (.025)	.240 (.025)	.125 (.018)	.162 (.016)	-	-
Pupil-Teacher Ratio	-.116 (.049)	-.103 (.049)	-.072 (.031)	-.072 (.031)	-.079 (.047)	-.129 (.046)
Teacher Salary	4.187E-5 (2.8E-5)	3.34E-5 (2.8E-5)	1.71E-5 (1.7E-5)	1.76E-5 (1.7E-5)	0.72E-5 (2.9E-5)	2.49E-5 (2.8E-5)

a. Also includes birth weight, first and second born dummies, dummies for mother’s age at birth 18-19 and 20-29, child’s age in months and its square, dummies for child age in years, mother’s schooling, race and sex dummies and cumulative family income.

Table 4b: Alternative Specifications of the “Hybrid” Production Function: Reading Percentile Score<sup>a</sup>  
(robust standard errors in parentheses)

	Contemporaneous	Cumulative	Value Added		Sibling FE	Child FE
			(1)	(2)		
<b>Home Inputs</b>						
(1) Home (0)	.132 (.018)	.063 (.017)	.087 (.016)	.052 (.017)	.083 (.018)	.027 (.018)
(2) Home (-1)						
Age 6-7	-	.130 (.028)	-	-	.012 (.031)	.011 (.031)
Age 8-13	-	.061 (.018)	-	.015 (.018)	.050 (.019)	.022 (.019)
(3) Home (-2)						
Age 8-9	-	.162 (.027)	-	.114 (.024)	.069 (.029)	.063 (.025)
Age 10-13	-	.057 (.021)	-	.045 (.018)	.058 (.022)	.025 (.021)
(4) Home (-3)						
Age 10-11	-	.082 (.032)	-	-	-.007 (.031)	-
Age 12-13	-	.004 (.032)	-	-	.025 (.030)	-
(5) Home (-4)						
Age 12-13	-	.120 (.036)	-	-	-	-
Lag Test Score	-	-	.452 (.013)	.447 (.013)	-	-
Mother’s AFQT	.276 (.020)	.264 (.024)	.171 (.019)	.168 (.019)	-	-
Pupil-Teacher Ratio	-.092 (.049)	-.081 (.049)	-.044 (.030)	-.043 (.030)	.068 (.048)	.046 (.047)
Teacher Salary	4.26E-5 (2.7E-5)	3.42E-5 (2.7E-5)	1.99E-5 (1.7E-5)	2.02E-5 (1.7E-5)	-0.13E-5 (2.9E-5)	2.63E-5 (2.8E-5)

a. Also includes birth weight, first and second born dummies, dummies for mother’s age at birth 18-19 and 20-29, child’s age in months and its square, dummies for child age in years, mother’s schooling, race and sex dummies and cumulative family income.

Table 5a  
 Cross-validation root-mean-squared error (RMSE) for Alternative Specifications  
 of the Production Function with Baseline Variables: Percentile and Raw Scores

	Random Holdout Sample <sup>a</sup>				Race x Sex Holdout Sample <sup>b</sup>			
	Math		Reading		Math		Reading	
	Percentile	Raw	Percentile	Raw	Percentile	Raw	Percentile	Raw
Contemporaneous	23.26	8.21	24.53	8.52	24.44	8.61	25.48	8.75
Cumulative	23.03	8.14	24.38	8.45	24.16	8.53	25.33	8.68
Value-added	23.06	8.15	24.30	8.43	24.24	8.57	24.37	8.70
Value-added plus Lags	23.00*	8.13*	24.30*	8.42*	24.02*	8.49*	25.20*	8.66*
Sibling Fixed Effects	23.05	8.15	24.44	8.46	24.67	8.70	26.13	8.85
Child Fixed Effects	28.24	10.15	25.32	8.75	34.83	11.34	30.55	9.79

a. Based on 6 random hold-out samples. Model is estimated on five of the six groups and used to generate RMSE for the left out groups. The number shown is the average RMSE based on three replications of this procedure.

b. Based on 6 race/sex groups. Model is estimated on one race-sex group (e.g., white boys) and used to generate the RMSE for other five race x sex groups.

\* - denotes specification with the smallest RMSE value.

Table 5b  
 Cross-validation root-mean-squared error (RMSE) for Alternative Specifications  
 of the “Hybrid“ Production Function: Percentile and Raw Scores

	Random Holdout Sample <sup>a</sup>				Race x Sex Holdout Sample <sup>b</sup>			
	Math		Reading		Math		Reading	
	Percentile	Raw	Percentile	Raw	Percentile	Raw	Percentile	Raw
Contemporaneous	23.00	8.12	24.38	8.47	24.35	8.57	25.39	8.71
Cumulative	22.84	8.07	24.26	8.41	24.11	8.50	25.28	8.65
Value-added	22.83	8.07	24.12*	8.38*	24.22	8.55	24.32	8.68
Value-added plus Lags	23.80*	8.06*	24.22	8.39	23.98*	8.47*	25.17*	8.66*
Sibling Fixed Effects	22.89	8.09	24.35	8.43	24.63	8.68	26.10	8.83
Child Fixed Effects	30.71	11.31	26.50	8.90	34.94	11.46	30.54	9.78

a. Based on 6 random hold-out samples. Model is estimated on five of the six groups and used to generate RMSE for the left out groups. The number shown is the average RMSE based on three replications of this procedure.

b. Based on 6 race/sex groups. Model is estimated on one race-sex group (e.g., white boys) and used to generate the RMSE for other five race x sex groups.

\* - denotes specification with the smallest RMSE value.

Table 6a  
 Black – White Gap Closed by Home Inputs and by Mother’s AFQT:  
 Value Added with Home Lags

	Actual Gap	Predicted Gap		Closed by Home		Closed by AFQT	
		(1)	(2)	(1)	(2)	(1)	(2)
<b>Math Raw</b>							
Score							
Boys	7.9	6.7	7.5	1.8	1.6	3.7	2.9
Girls	6.3	5.7	6.5	1.7	1.5	3.7	2.9
<b>Reading Raw</b>							
Score							
Boys	7.0	6.3	6.6	1.8	1.6	3.9	3.5
Girls	5.6	5.5	5.8	1.6	1.5	3.9	3.5
<b>Math Percentile</b>							
Score							
Boys	22.1	18.7	21.2	4.7	4.1	10.1	8.0
Girls	17.6	15.9	18.3	4.3	3.8	10.1	8.0
<b>Reading Percentile</b>							
Score							
Boys	18.0	16.4	17.6	4.6	4.1	9.7	8.6
Girls	15.2	14.4	15.5	4.2	3.7	9.7	8.6

Table 6b  
Hispanic – White Gap Closed by Home Inputs and by Mother’s AFQT:  
Value Added with Home Lags

	Actual Gap	Predicted Gap		Closed by Home		Closed by AFQT	
		(1)	(2)	(1)	(2)	(1)	(2)
<b>Math Raw</b>							
Score							
Boys	5.6	5.5	5.9	1.7	1.5	3.1	2.5
Girls	5.7	4.9	5.2	1.2	1.1	3.1	2.5
<b>Reading Raw</b>							
Score							
Boys	4.5	4.8	4.5	1.7	1.5	3.3	3.0
Girls	3.7	4.1	3.7	1.2	1.1	3.3	3.0
<b>Math Percentile</b>							
Score							
Boys	14.1	14.0	15.3	4.5	3.9	8.6	6.8
Girls	15.6	13.3	14.3	3.2	2.8	8.6	6.8
<b>Reading Percentile</b>							
Score							
Boys	11.3	11.7	11.4	4.3	3.8	8.3	7.3
Girls	10.2	11.1	10.4	3.1	2.7	8.3	7.3



Figure 1a: Comparison of PIAT-Reading Percentile Scores by Age by Race/Ethnicity

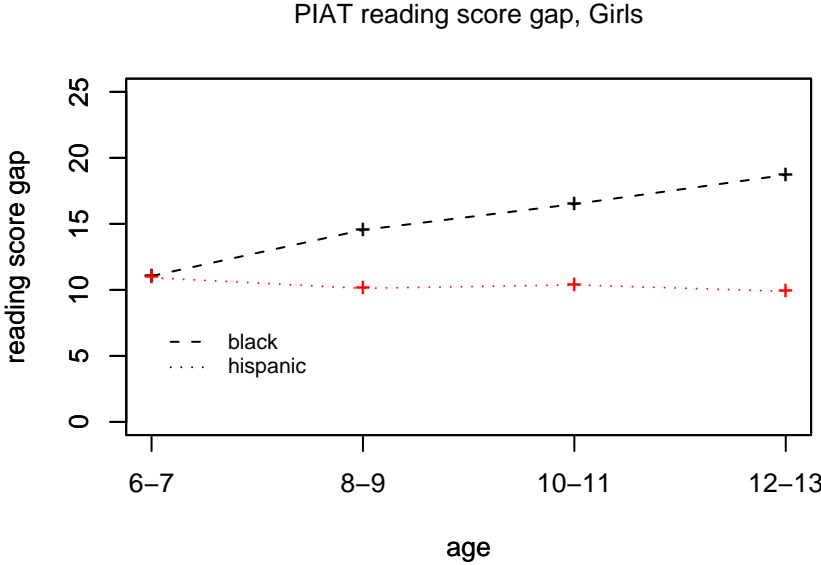
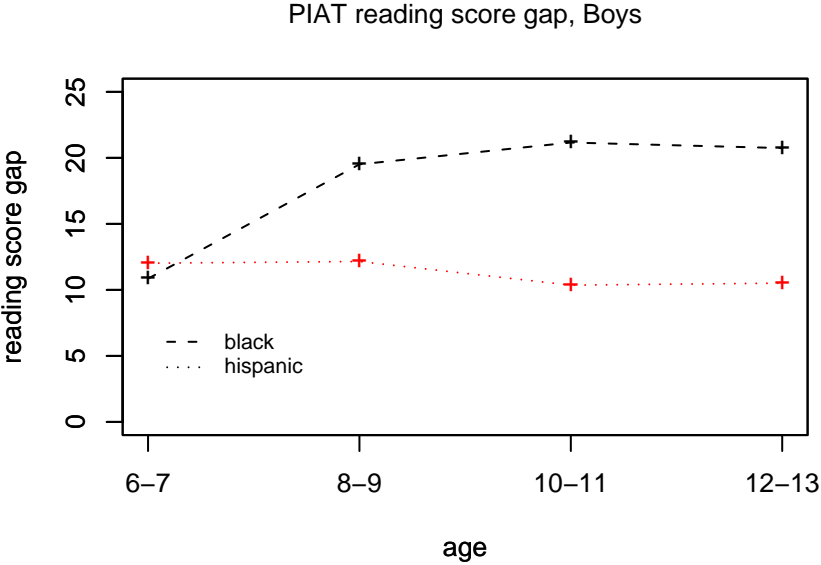
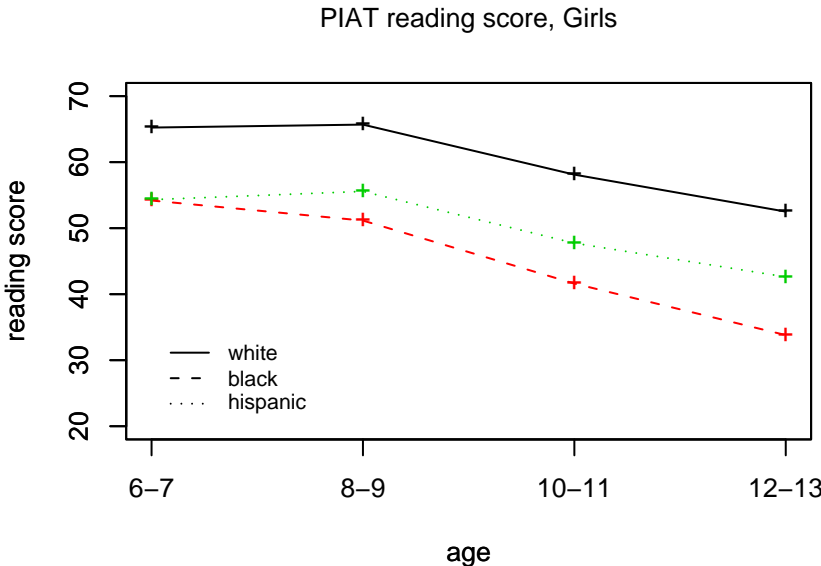
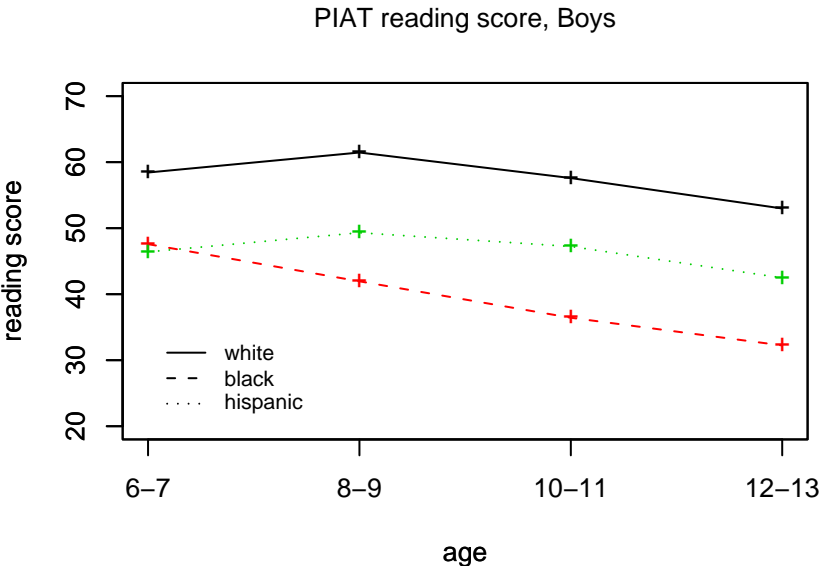


Figure 1b: Comparison of PIAT–Math Percentile Scores by Age by Race/Ethnicity

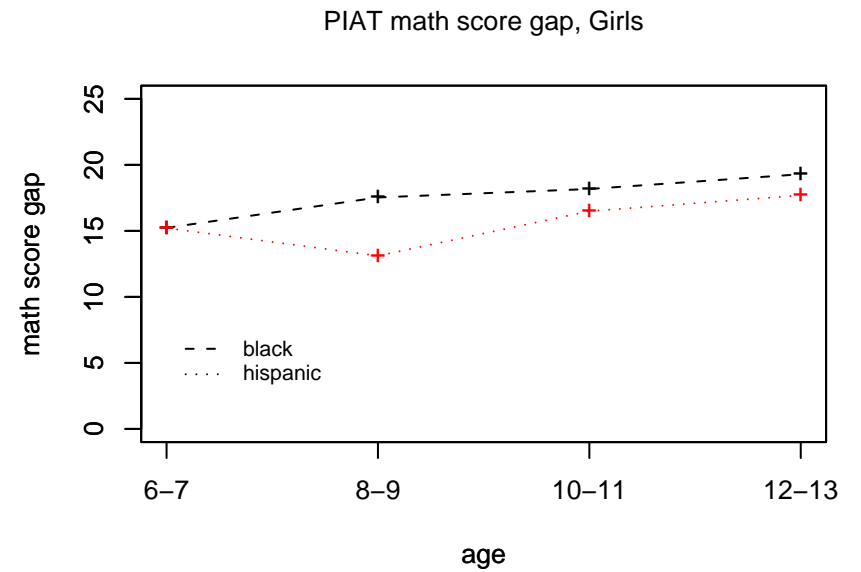
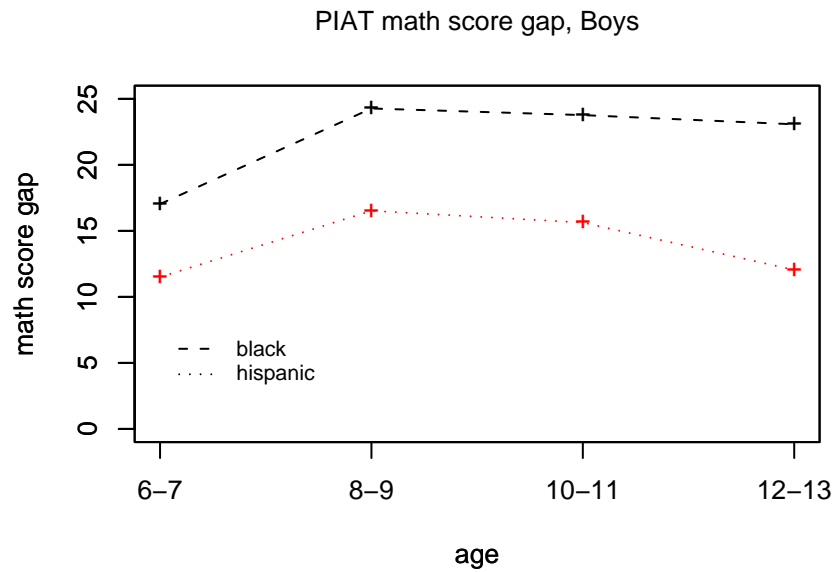
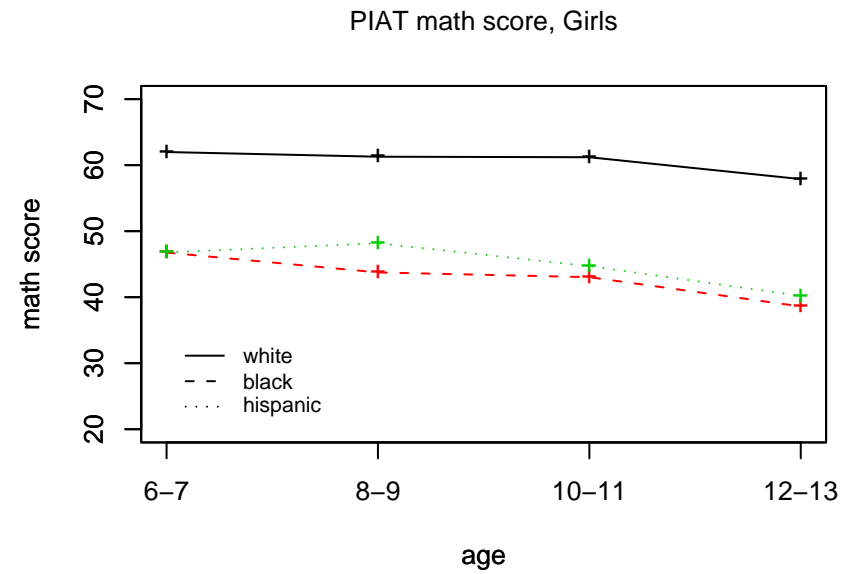
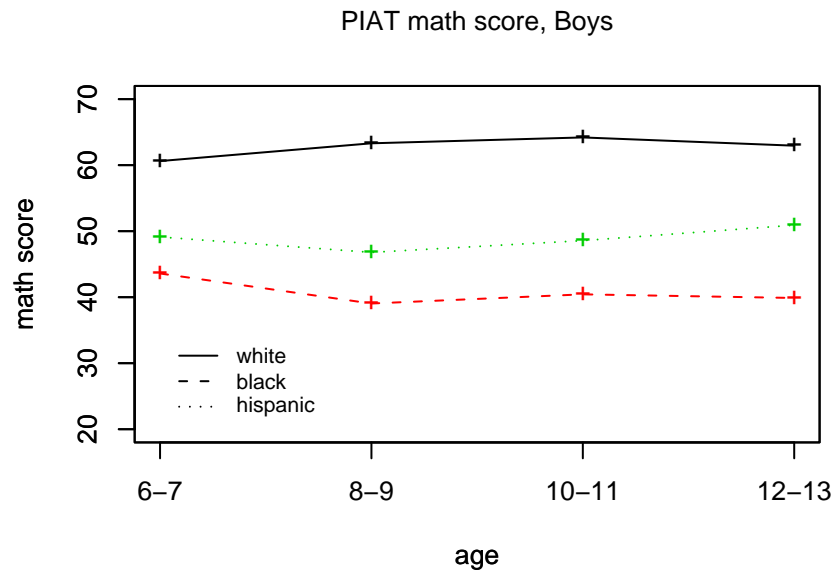


Figure A.1a: Comparison of PIAT-Reading Raw Scores by Age by Race/Ethnicity

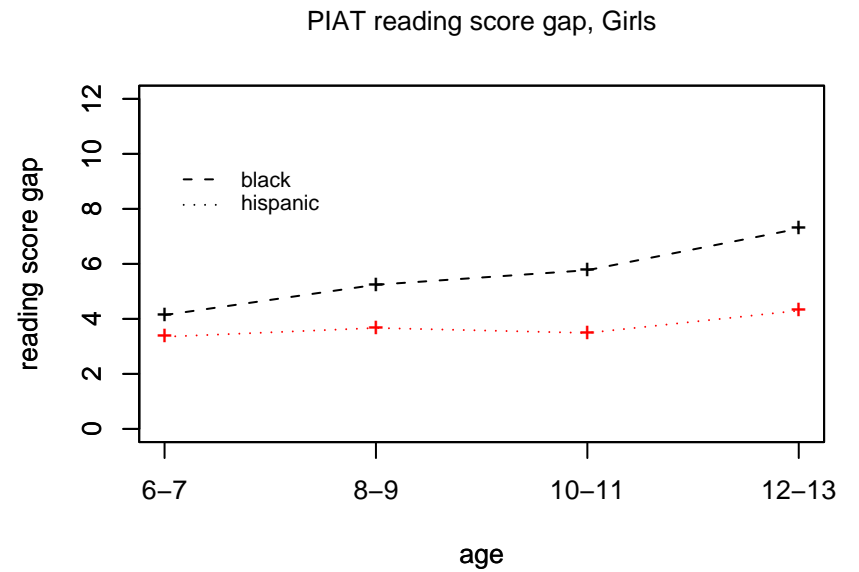
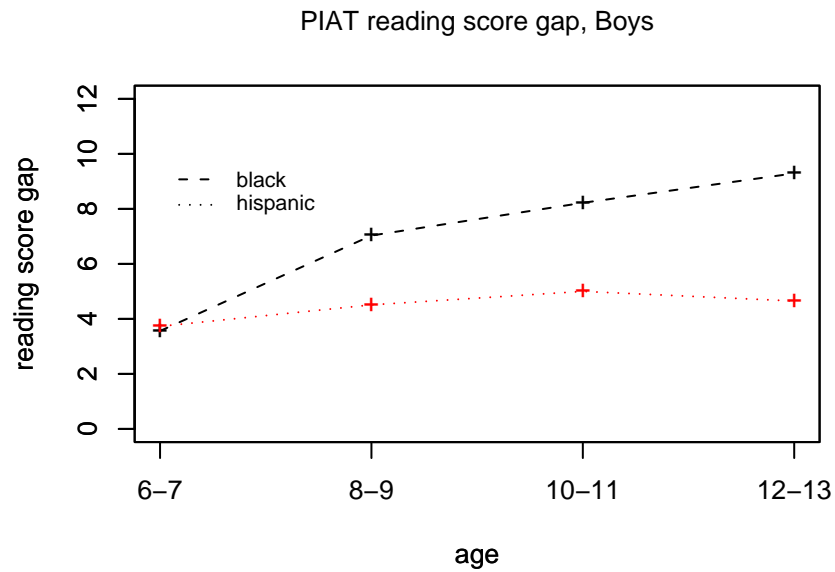
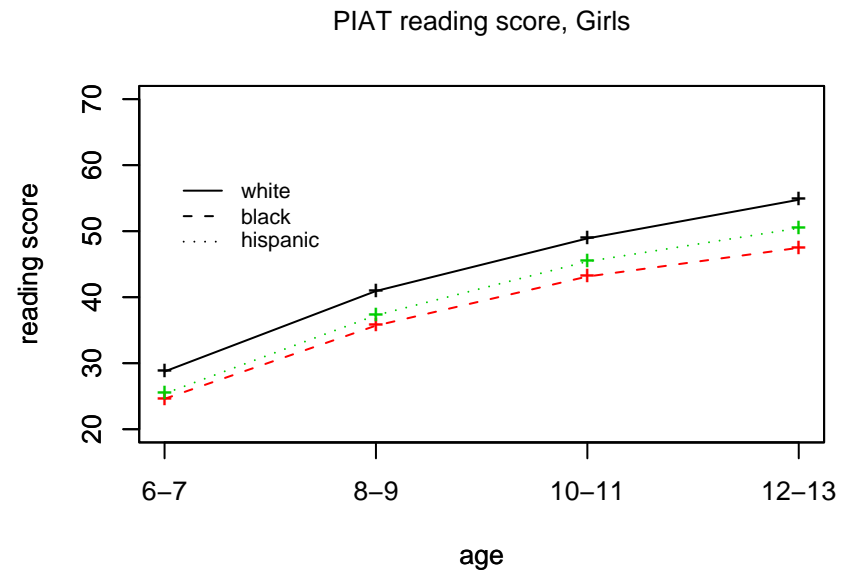
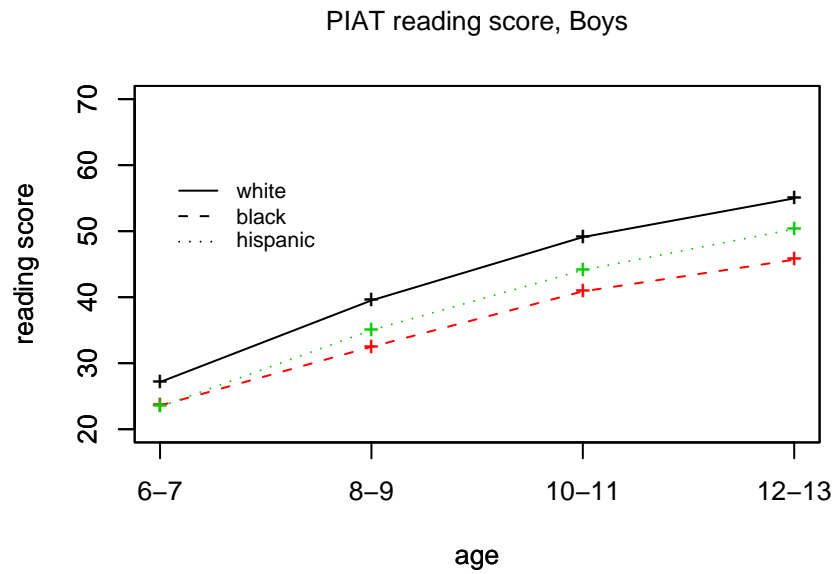


Figure A.1b: Comparison of PIAT–Math Raw Scores by Age by Race/Ethnicity

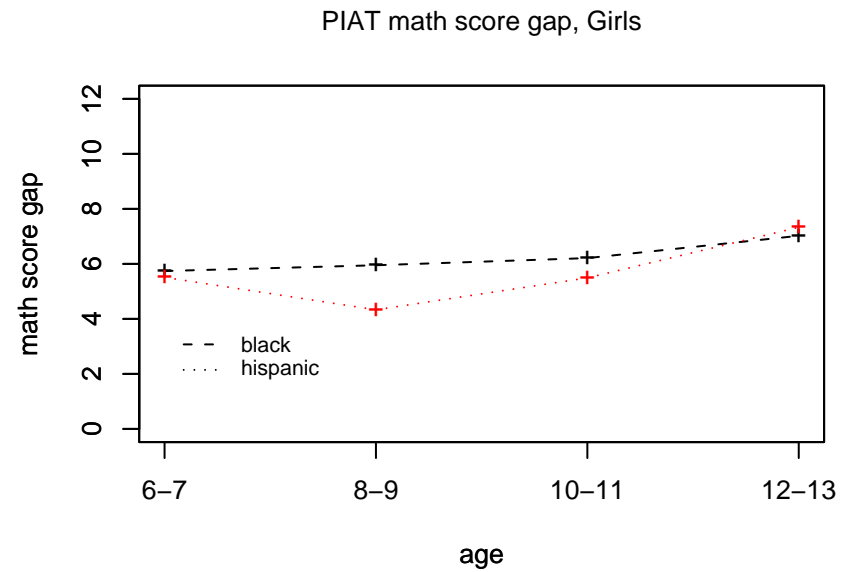
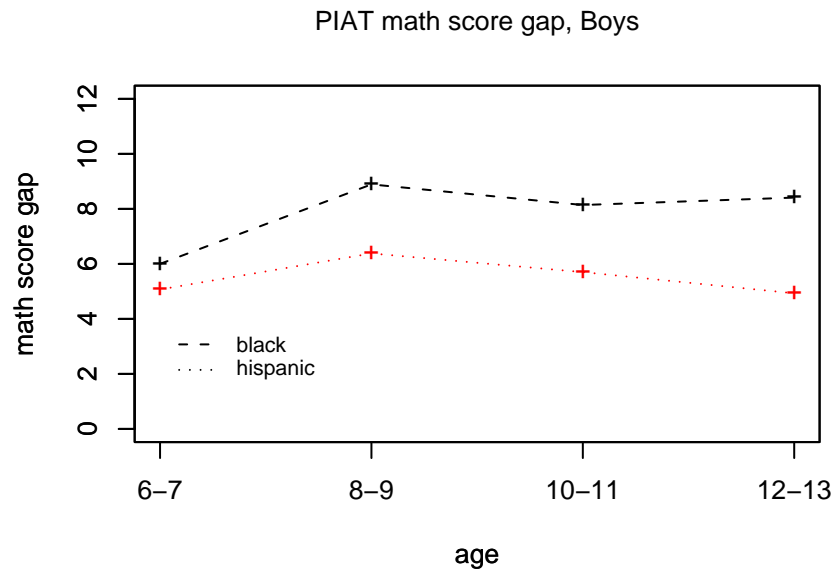
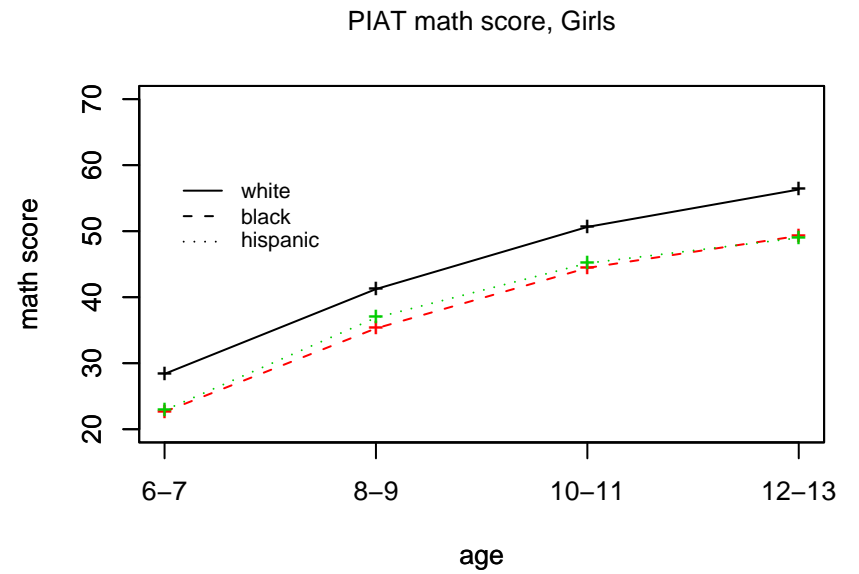
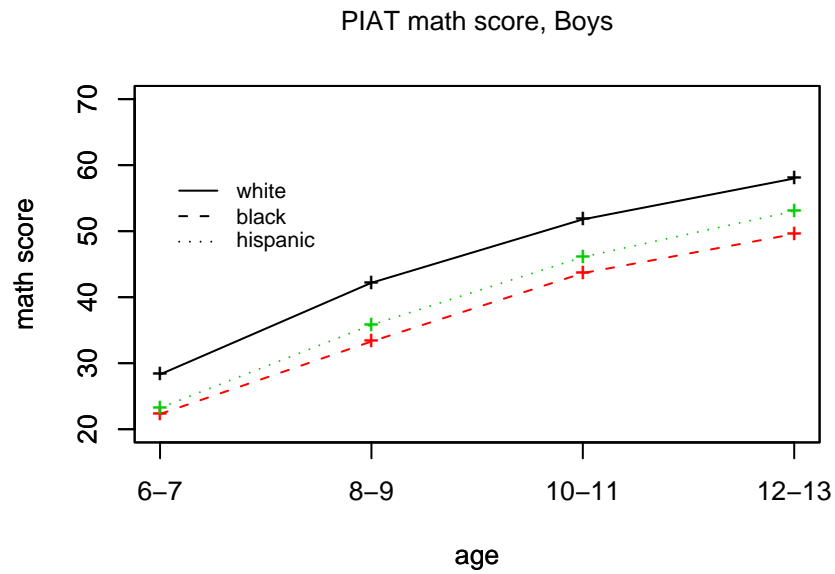
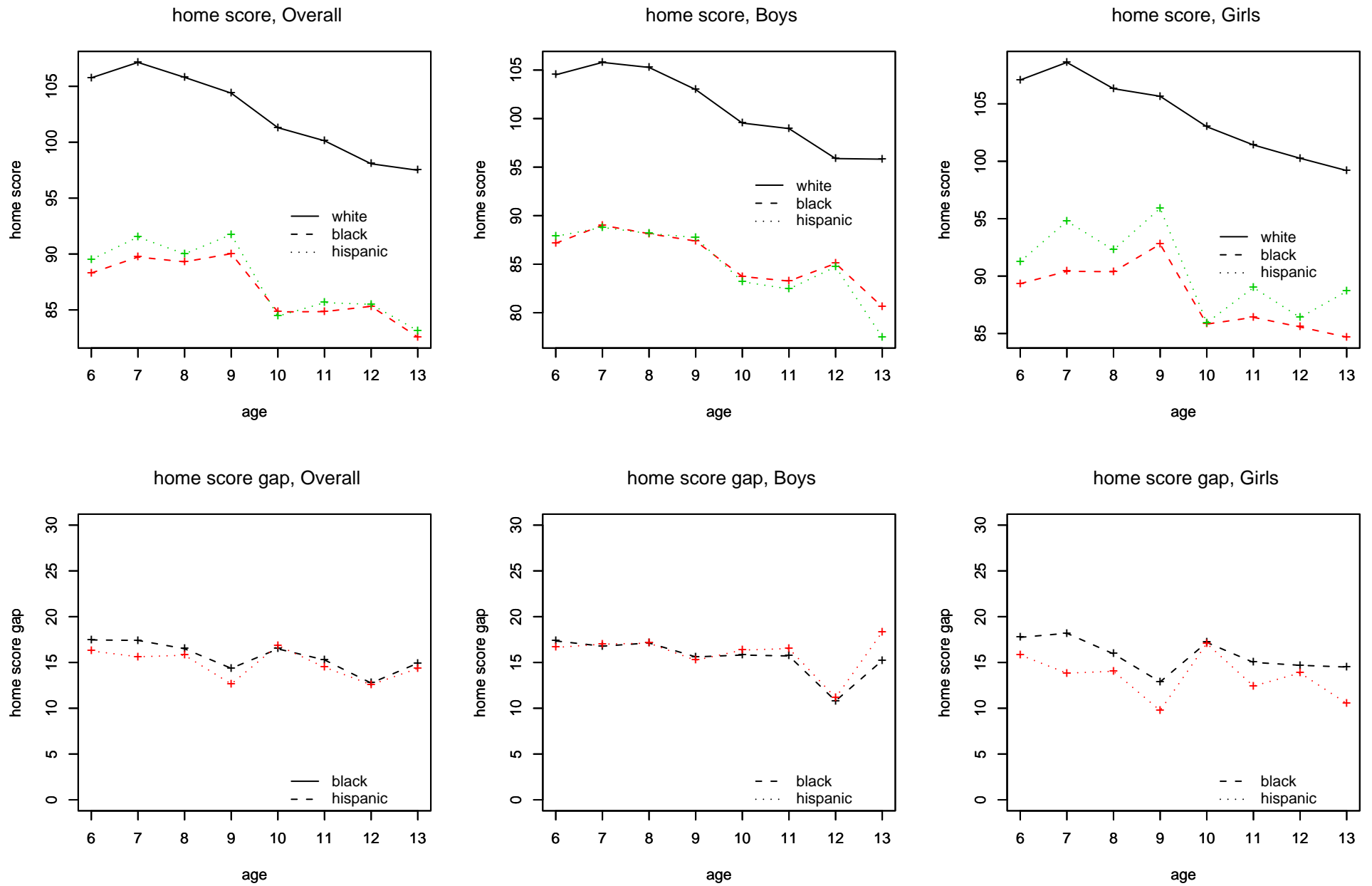
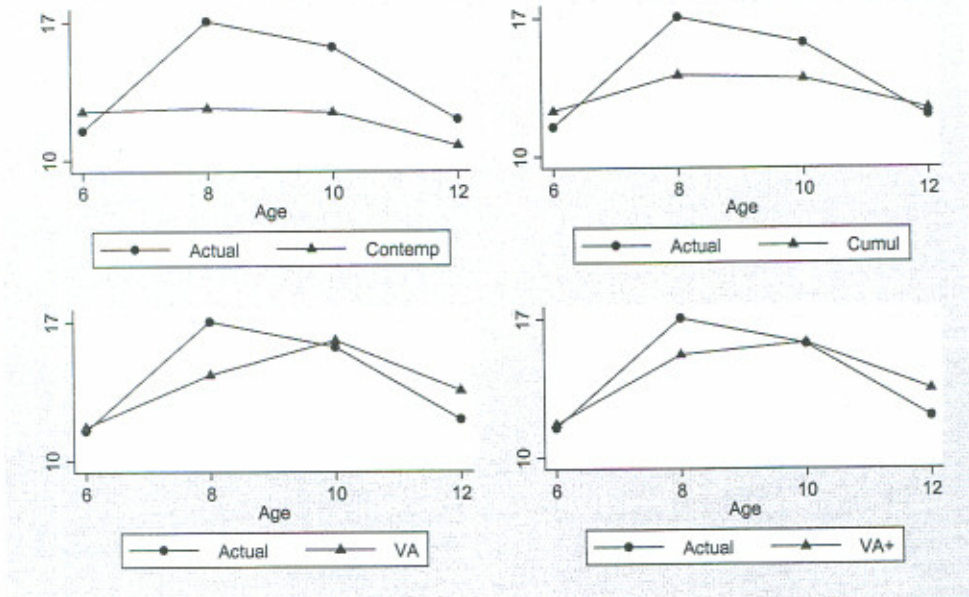


Figure 2: Comparison of Current Home Score by Age by Race/Ethnicity

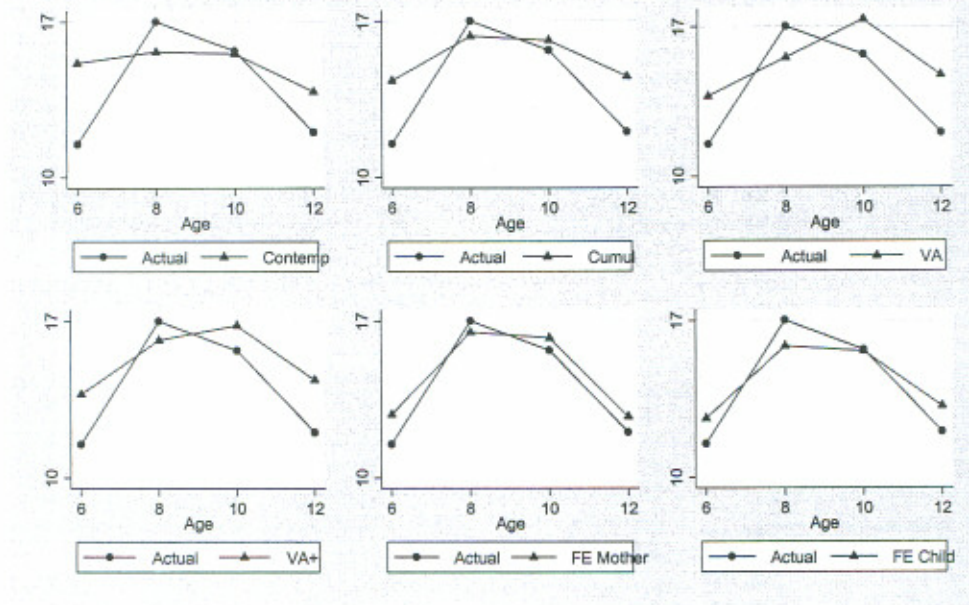




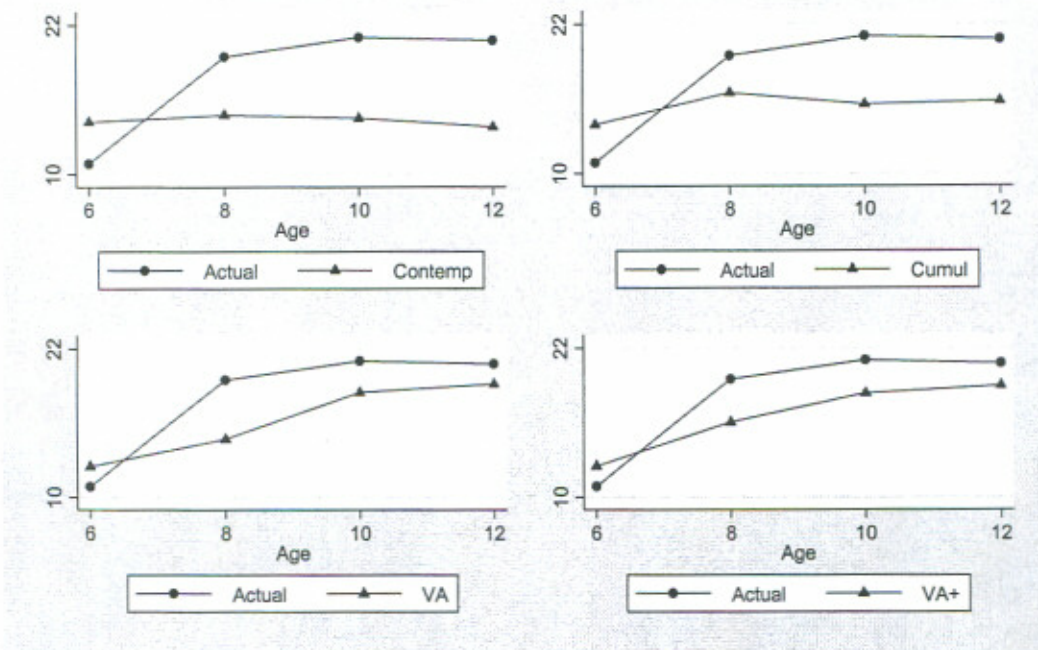
## Pred. vs. Act. Hispanic-White Difference in Piat Math Perc. Scores for Boys Alternative Specifications of the Production Function



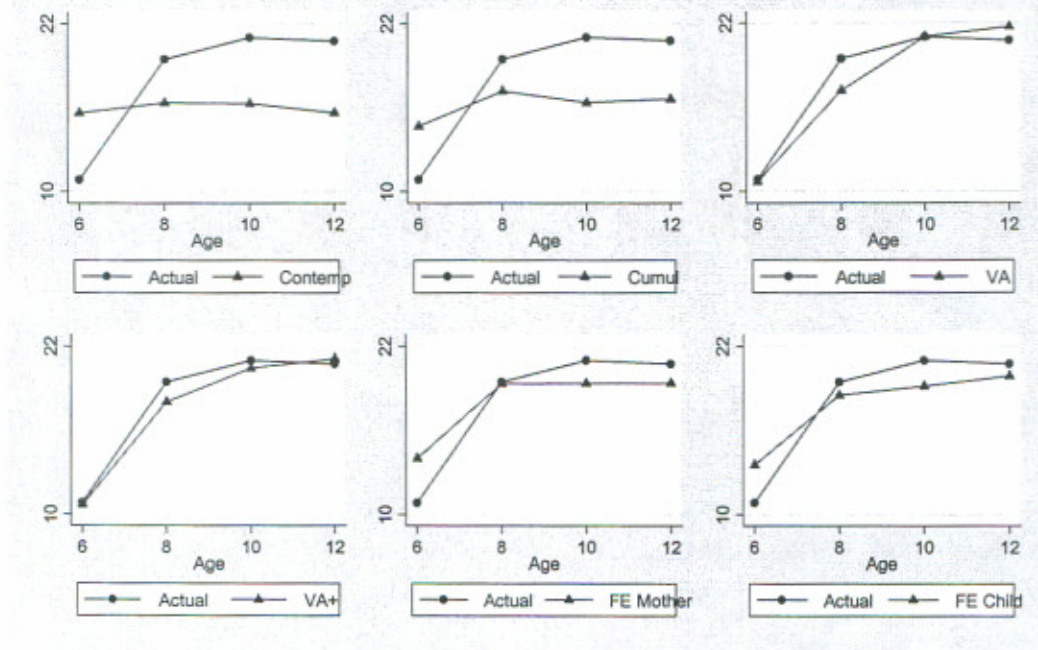
## Pred. vs. Act. Hispanic-White Difference in Piat Math Perc. Scores for Boys Alternative Specifications of the Hybrid Production Function



## Pred. vs. Act. Black-White Difference in Piat Reading Perc. Scores for Boys Alternative Specifications of the Production Function

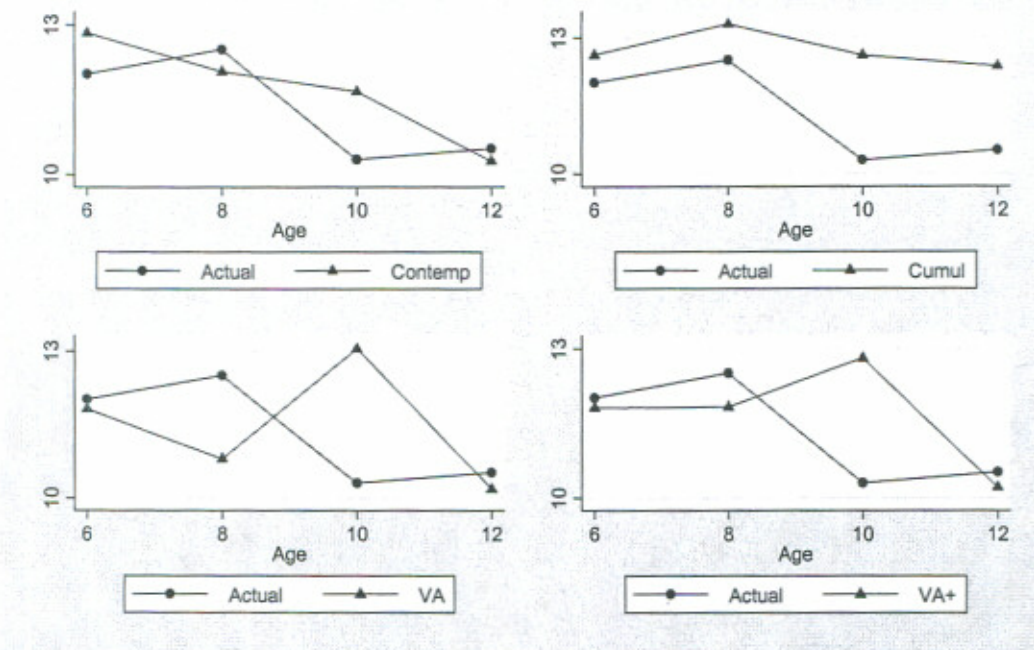


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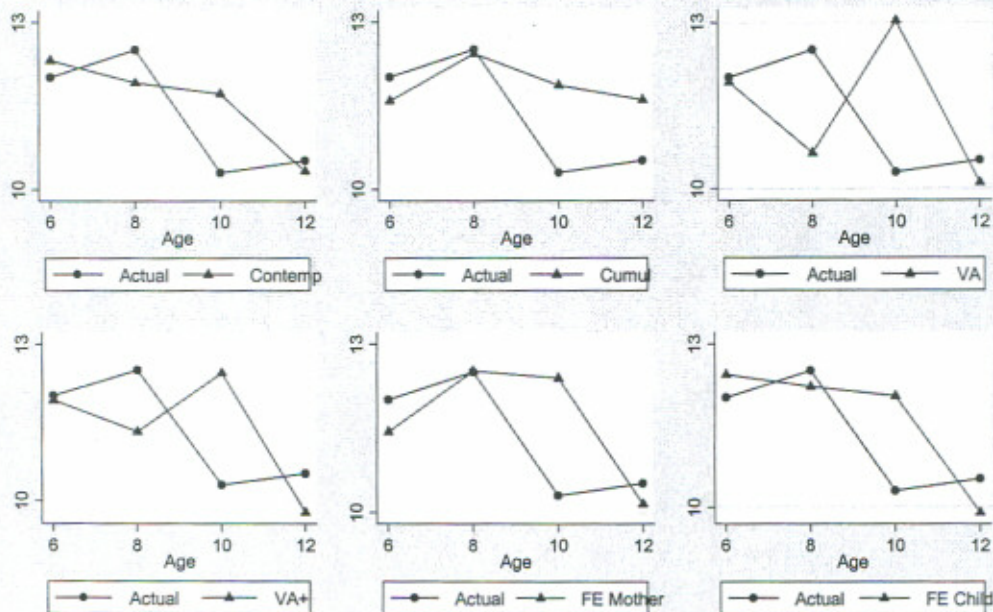




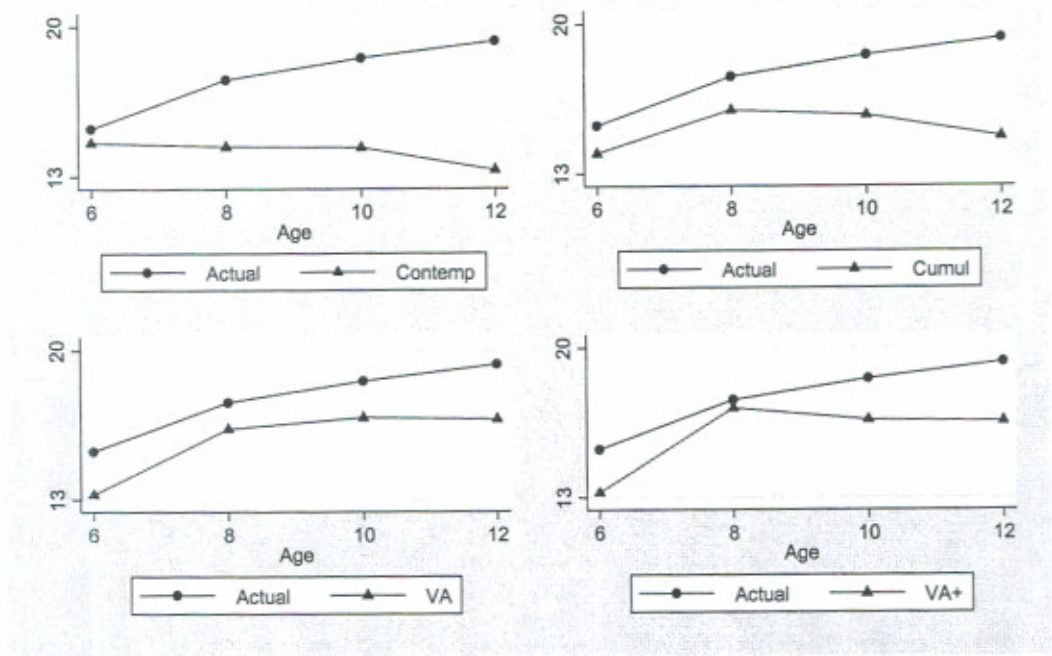
Pred. vs. Act. Hispanic-White Difference in Piat Reading Perc. Scores for Boys  
Alternative Specifications of the Production Function



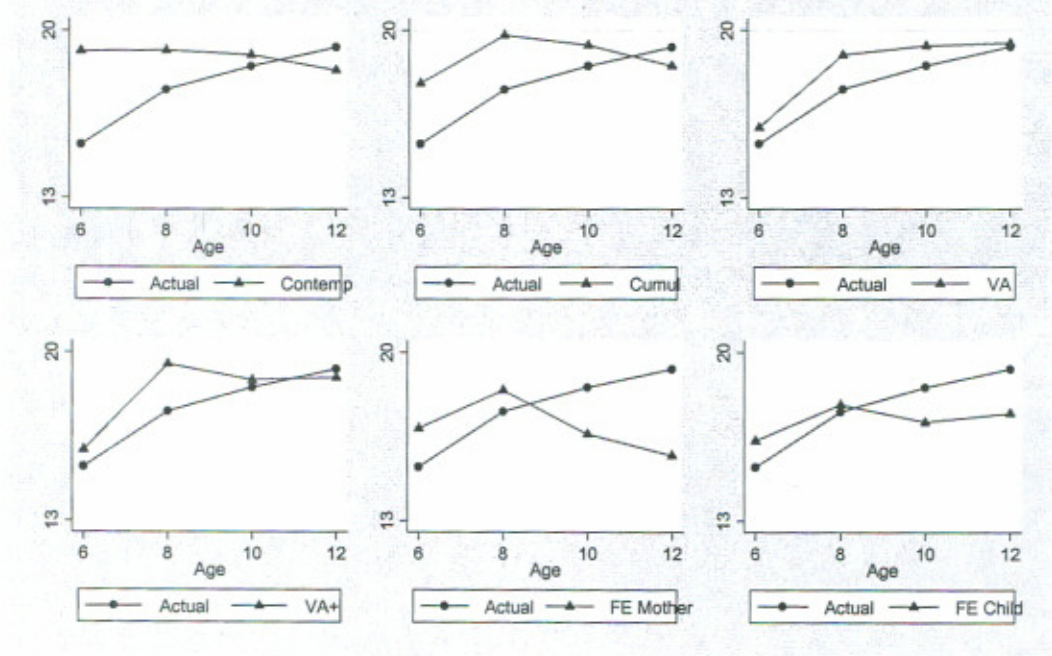
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Alternative Specifications of the Hybrid Production Function



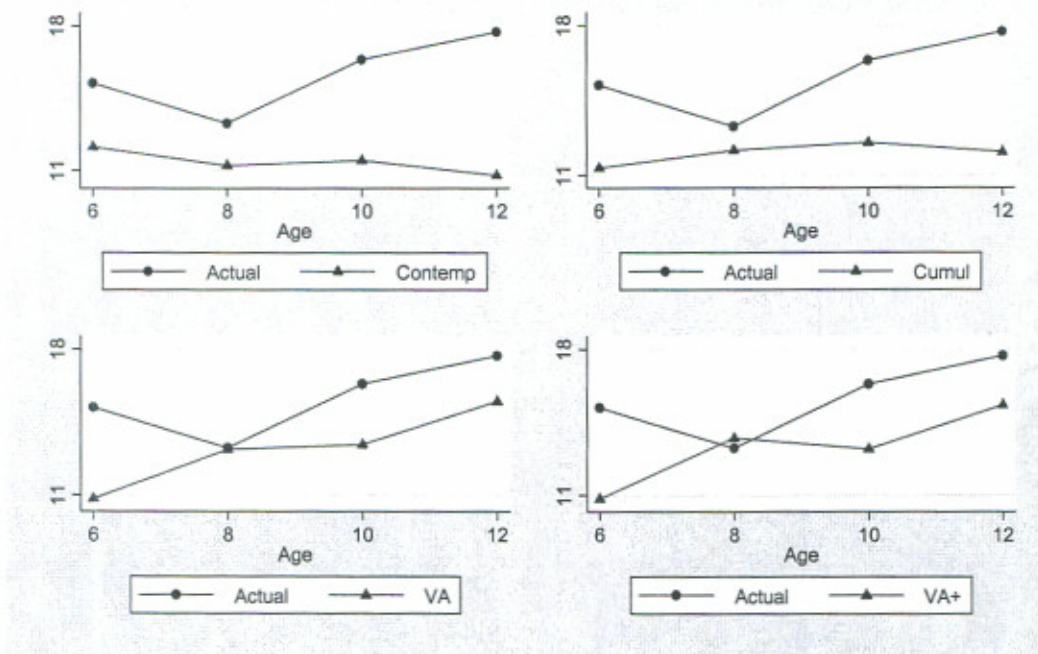
## Pred. vs. Act. Black-White Difference in Piat Math Perc. Scores for Girls Alternative Specifications of the Production Function



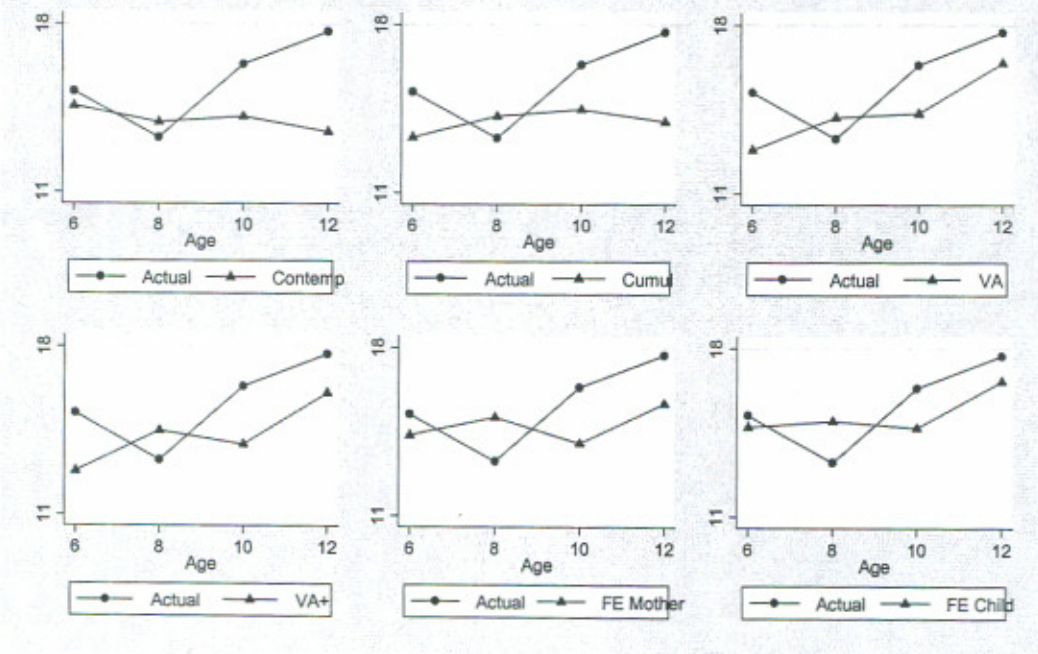
## Pred. vs. Act. Black-White Difference in Piat Math Perc. Scores for Girls Alternative Specifications of the Hybrid Production Function



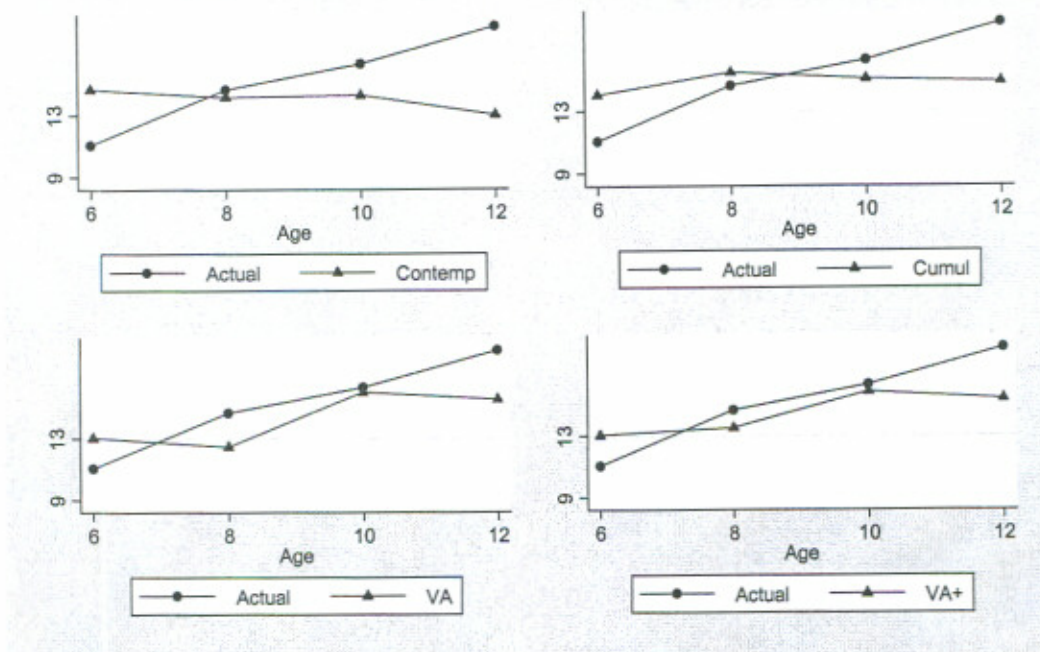
Pred. vs. Act. Hispanic-White Difference in Piat Math Perc. Scores for Girls  
 Alternative Specifications of the Production Function



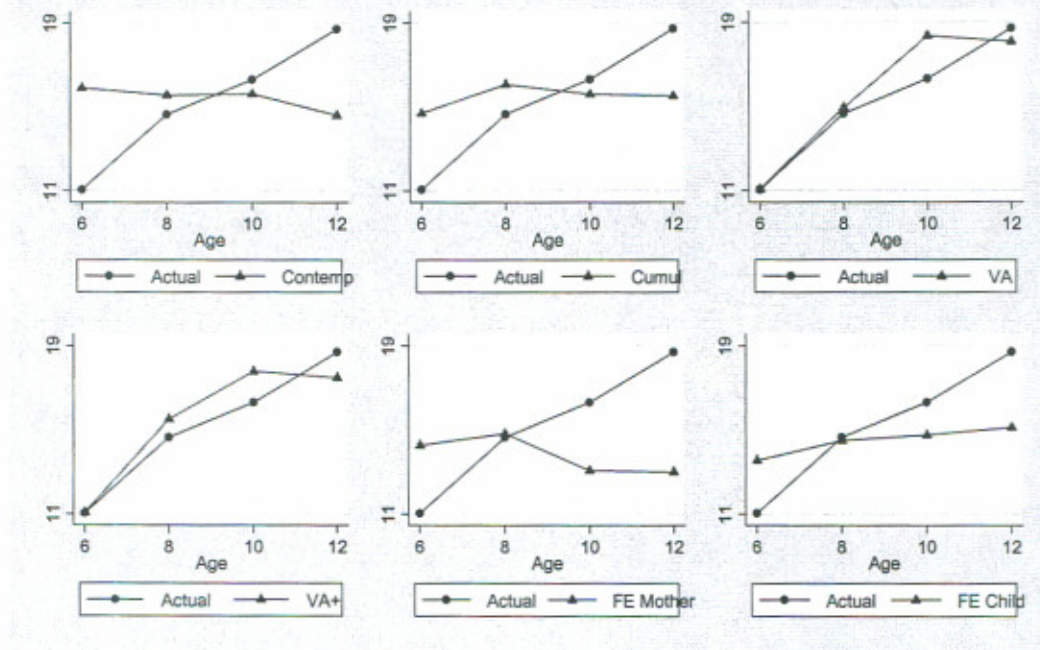
Pred. vs. Act. Hispanic-White Difference in Piat Math Perc. Scores for Girls  
 Alternative Specifications of the Hybrid Production Function



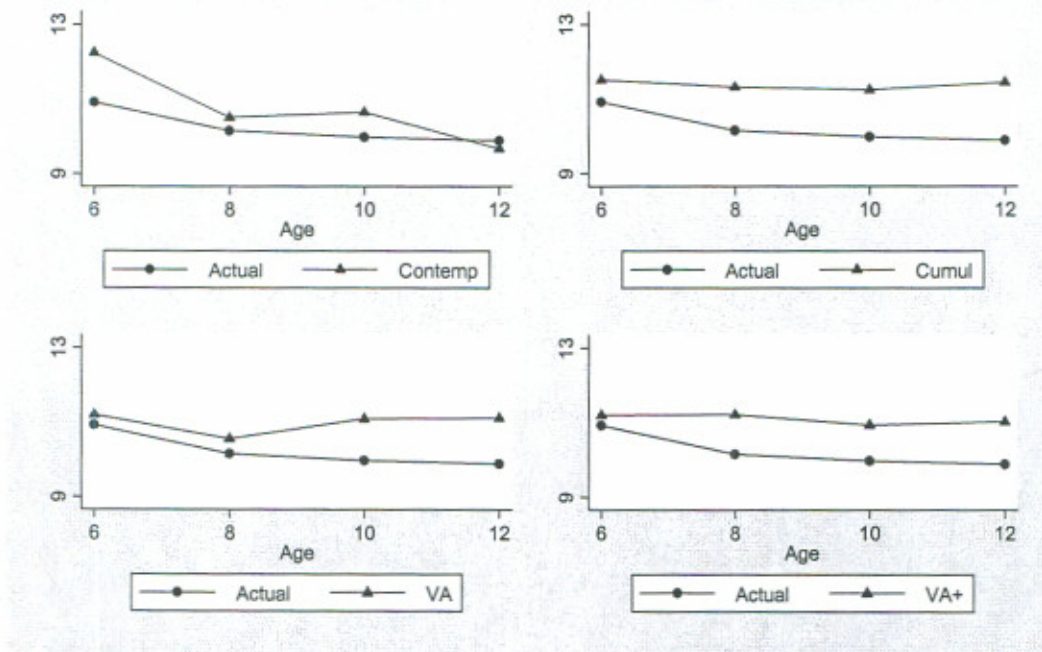
## Pred. vs. Act. Black-White Difference in Piat Reading Perc. Scores for Girls Alternative Specifications of the Production Function



## Pred. vs. Act. Black-White Difference in Piat Reading Perc. Scores for Girls Alternative Specifications of the Hybrid Production Function



## Pred. vs. Act. Hispanic-White Difference in Piat Reading Perc. Scores for Girls Alternative Specifications of the Production Function



## Pred. vs. Act. Hispanic-White Difference in Piat Reading Perc. Scores for Girls Alternative Specifications of the Hybrid Production Function

